

Optimal policy learning using Stata

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Optimal treatment assignment of a thresholdbased policy: empirical protocol and related issues

Giovanni Cerulli 🔀 🗓

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INTRODUCTION

- 1. This paper deals with ex-ante data-driven optimal design of (micro) policies
- 2. It is embedded within the **optimal policy learning (OPL)** literature
- 3. It contributes by stressing the policymaker perspective
- 4. It suggests a **menu strategy** to deal with optimal solution's *monotonicity*



OPTIMAL POLICY LEARNING - 1

Optimal policy learning

Frontier of the "econometrics of program evaluation"

Changing policy perspective

From policy "ex-post" evaluation to "ex-ante" optimal policy design

Prediction based

Compared to ex-post evaluation (based on inference), OPL targets optimal "prediction", entailing a central role of "machine learning"





DEFINITION OF OPL

What is policy learning?

Process of improving program welfare achievements by re-iterating similar policies over time

Optimal treatment assignment

Policymakers can optimally fine-tune the treatment assignment of a prospective policy using the results from an RCT or observational study. Assignment rules depends on the class of policies considered (here we focus on threshold-based and linear-combination policies)

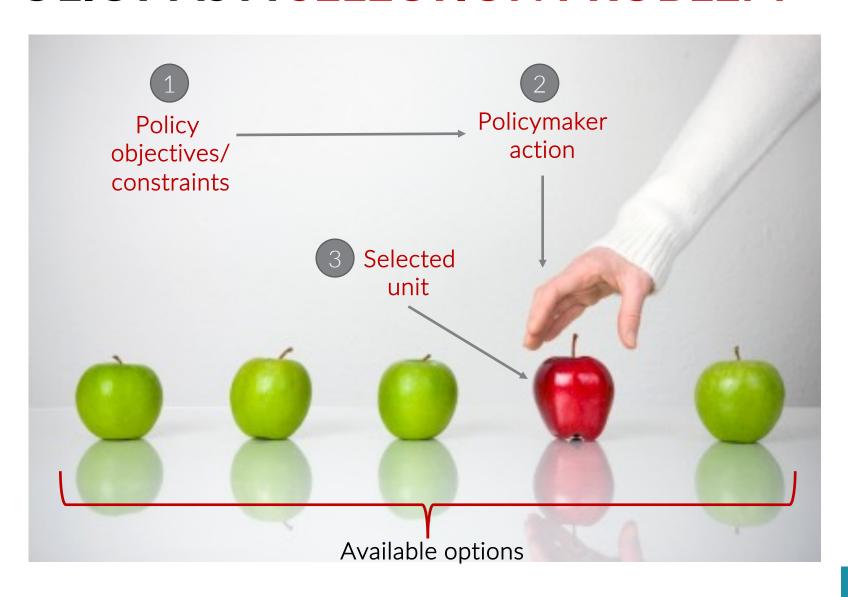
Maximizing constrained welfare

The policymaker hardly manage to reach the best solution (unconstrained maximum welfare) because of institutional/economic contains of various sort





POLICY AS A SELECTION PROBLEM





STATE-OF-THE ART - 1

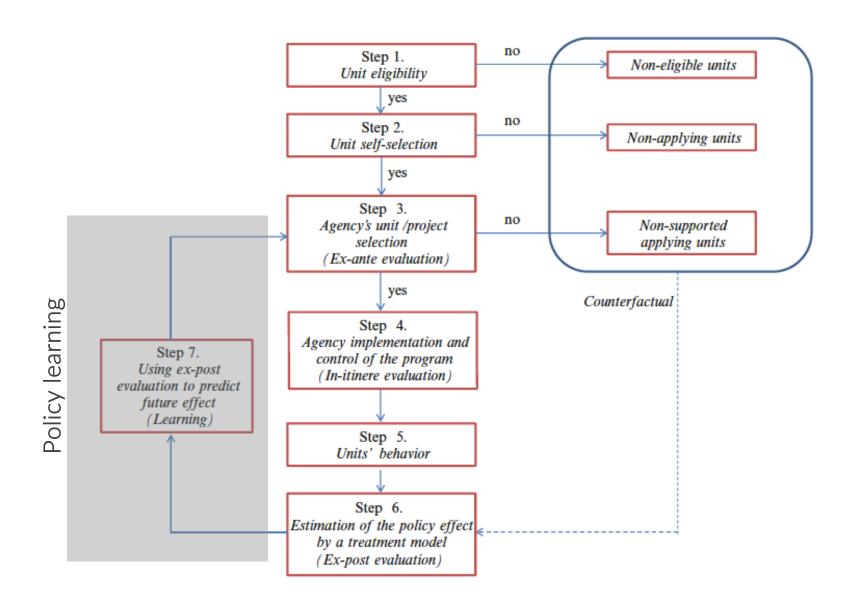
Manski C.F. (2004), Statistical Treatment Rules for Heterogeneous Populations, *Econometrica*, 72, 4, 1221–1246.

Kitagawa T., Tetenov A. 2018. Who should be treated? empirical welfare maximization methods for treatment choice, *Econometrica*. 86, 2, 591–616.

Bhattacharya D., Dupas P. 2012. Inferring Welfare Maximizing Treatment Assignment under Budget Constraints. *Journal of Econometrics*, 167, 1, 168–196.

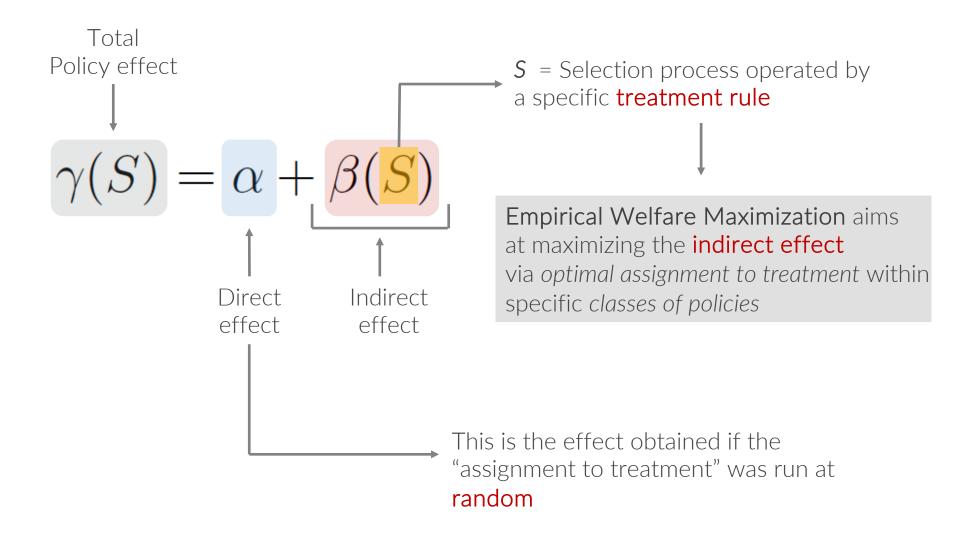


POLICY LEARNING WITHIN THE POLICY EVALUATION CYCLE





POLICY DIRECT AND INDIRECT EFFECT



OPTIMAL TREATMENT ASSIGNMENT - 1

Let X be an individual's vector of characteristics, Y an outcome of interest, $T = \{0, 1\}$ a binary treatment. A policy assignment rule \mathcal{G} is a function mapping X to T, specifying which individuals are or are not to be treated:

$$\mathcal{G}:X\to T$$

Define the (population) policy conditional average treatment effect as:

$$\tau(X) = E(Y_1|X) - E(Y_0|X)$$

where Y_1 and Y_0 represent the two potential outcomes of the policy, and $E_X[\tau(X)] = \tau$ the average treatment effect.



OPTIMAL TREATMENT ASSIGNMENT - 2

Under selection-on-observables, we know that:

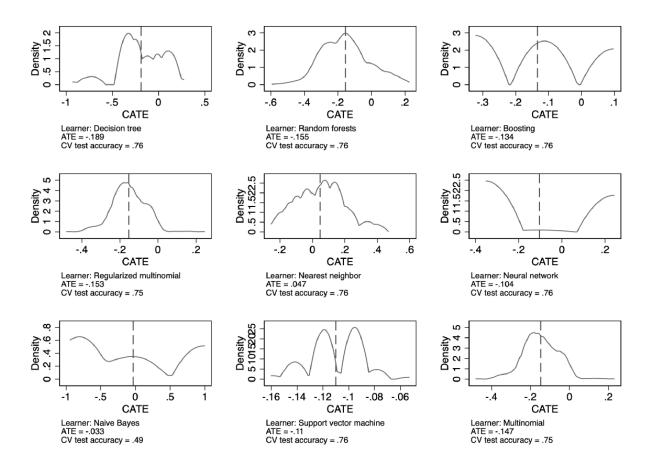
$$\tau(X) = E(Y|X, T = 1) - E(Y|X, T = 0)$$

These two conditional expectations are identified by data. Whatever ML algorithm can be used for estimation (Boosting, Random forests, Neural networks, Nearest neighbor, etc.)

Extension to selection-on-unobservables straightforward



ML ESTIMATION OF $\tau(X)$



Estimation of the distribution of the conditional average treatment effects (CATE) using the ML methods implemented via c_ml_stata_cv (Cerulli, 2022). Note: dashed vertical line indicates the average treatment effect (ATE).

OPTIMAL TREATMENT ASSIGNMENT - 3

The estimated policy actual total effect (or welfare)

$$\widehat{W} = \sum_{i=1}^{N} T_i \cdot \widehat{\tau}(X_i)$$

and the estimated policy unconstrained optimal total effect (or unconstrained maximum welfare) as:

$$\widehat{W}^* = \sum_{i=1}^N \widehat{T}_i^* \cdot \widehat{\tau}(X_i)$$

where:

$$\hat{T}_i^* = \mathbf{1}[\hat{\tau}(X_i) > 0]$$

is the estimated optimal unconstrained policy assignment.

The difference between the estimated (unconstrained) maximum achievable welfare and the estimated welfare associated to the policy actually run is called *regret*, and it is defined as:

$$\widehat{regret} = \widehat{W}^* - \widehat{W}$$



EXAMPLE

Example of an optimal policy assignment rule The regret of this policy is equal to 16 = 26 - 10

ID	T	$\tau(X)$	$T \cdot \tau(X)$	T^*	$T^* \cdot \tau(X)$
1	1	9	9	1	9
2	1	-4	-4	0	0
3	1	5	5	1	5
4	0	6	0	1	6
5	0	-2	0	0	0
6	0	6	0	1	6
			10		26

Actual welfare reached

Maximum welfare feasible

regret
$$\longrightarrow$$
 26 – 10 = 16



NAÏVE OPTIMAL SELECTION

1. Given $\{X,Y,T\}$ from an already-implemented policy: estimate the idiosyncratic effect $\tau(X)$. This means we have learnt the mapping:

$$X \rightarrow \tau(X)$$
 (learning from experience)

- 2. Consider a prospective second policy round with a new eligible set $\{X'\}$, and compute the learnt $\{\tau(X')\}$ over X'.
- 3. Rank individuals so that: $\tau(X_1') > \tau(X_2') > \tau(X_3') > \dots > 0$.
- 4. Given a monetary budget C and a unit cost c_i , find N_1^* :

$$\sum_{i=1}^{N_1^*} c_i = C$$



OPTIMAL CONSTRAINED ASSIGNMENT

- Eligibility, budget, ethical, or institutional constrains make policymakers unable to implement the *optimal unconstrained* policy assignment
- ☐ They are obliged to rely on a <u>constrained assignment rule</u> selecting treated units according to their characteristics
- ☐ The welfare thus obtained may drop down
- Policymakers can however produce the largest feasible constrained welfare



EXAMPLE OF CONSTRAINED ASSIGNMENT: UNIVARIATE THRESHOLD-BASED POLICY

- The policymaker wants to treat only "young" people
- In theory, he can continue to use the naïve approach, by excluding from treatment all the individuals with age smaller than a certain age A*
- The problem is that different A* can induce different level of welfare
- The problem becomes that of choosing A* to maximize the effect/welfare



POLICY CLASSES

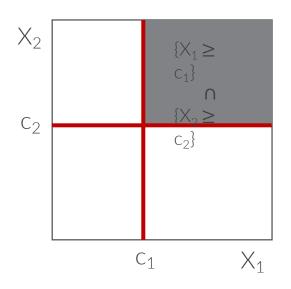
There exist however several classes of policies used by policymakers to select in a constrained decision context. The most popular are:

- ☐Threshold-based
- ☐ Linear combination
- ☐ Fixed-depth decision trees

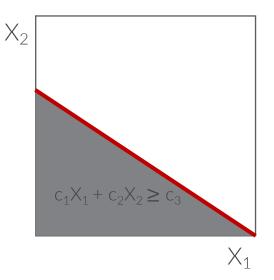


POLICY CLASSES (DECISION BOUNDARIES)

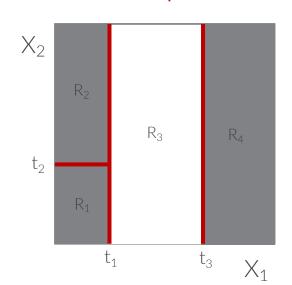
Threshold-based



Linear combination



Fixed-depth tree



 $X_1 < t_1$

 R_3

Legend:

—— Decision boundary

Selection area





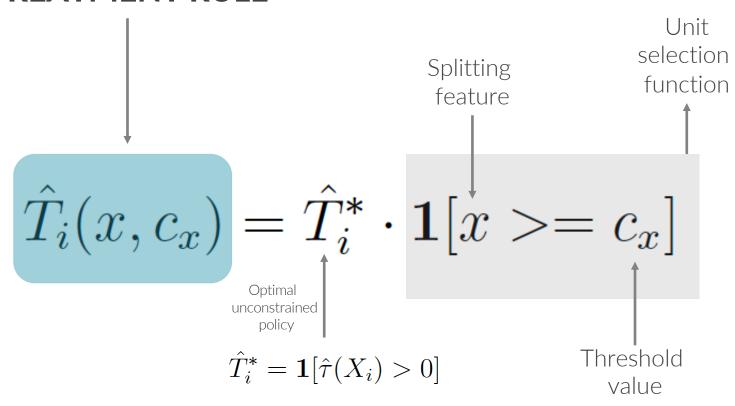
 $X_1 < t_3$

 R_4



Threshold-based policy

OPTIMAL CONSTRAINED TREATMENT RULE





OPTIMAL CONSTRAINED WELFARE

The corresponding welfare is a function of c_x :

$$\widehat{W}(x, c_x) = \sum_{i=1}^{N} \widehat{T}_i(x, c_x) \cdot \widehat{\tau}(X_i)$$

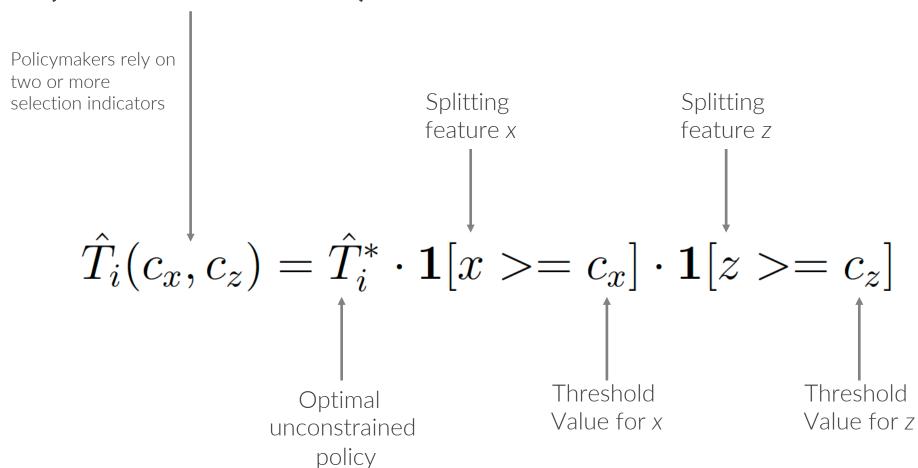
We define the optimal choice of the threshold c_x as the one maximizing $\widehat{W}(x, c_x)$ over c_x :

$$c_x^* = \mathrm{argmax}_{c_x}[\widehat{W}(x, c_x)]$$

If c_x^* exists, the estimated optimal constrained welfare will thus be equal to $\widehat{W}(c_x^*)$.



OPTIMAL CONSTRAINED TREATMENT RULE (MULTIVARIATE CASE)



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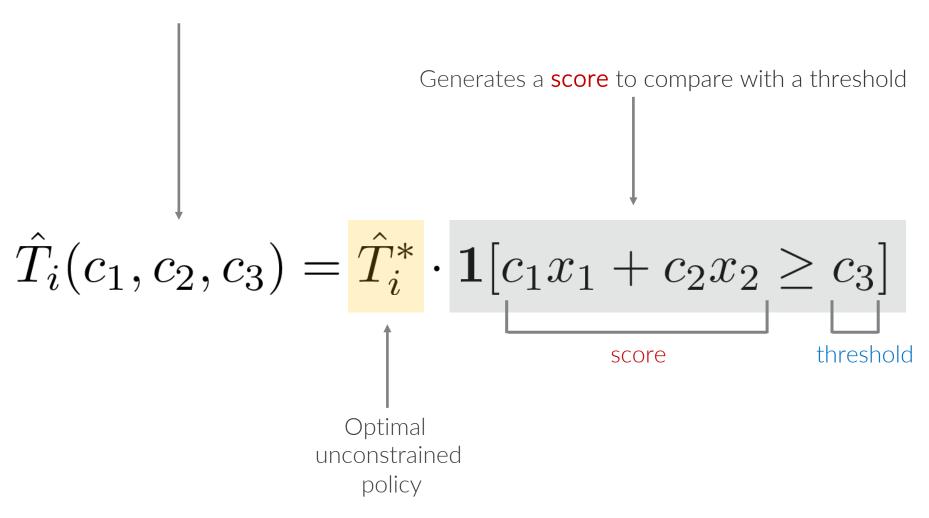
ESTIMATION

Procedure. Threshold-based optimal policy assignment

- 1. Suppose to have data from an RCT or from an observational study consisting of the information triple (Y, X, T) available for every unit involved in the program.
- 2. Run a quasi-experimental method with observable heterogeneity, estimate $\tau(X)$, and compute the (estimated) actual total welfare of the policy \widehat{W} .
- 3. Identify the estimated optimal unconstrained policy \hat{T}^* , and compute \widehat{W}^* , i.e. the estimated maximum total welfare achievable by the policy, and estimate the regret as $\widehat{W}^* \widehat{W}$.
- 4. Consider an estimated constrained selection rule $\hat{T}(x,c)$ based on a given set of selection variables, x, and related thresholds, c, and define the estimated maximum constrained welfare as $\widehat{W}(x,c)$.
- 5. Build a greed of K possible values for $c \in \{c_1, ..., c_K\}$, compute the optimal vector of thresholds c_{k^*} and the corresponding maximum estimated welfare $\widehat{W}(x, c_{k^*})$ thus achieved.



LINEAR COMBINATION POLICY (BIVARIATE CASE)



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APPLICATION

DATA: National Supported Work Demonstration (NSWD), an RCT by LaLonde (1986).

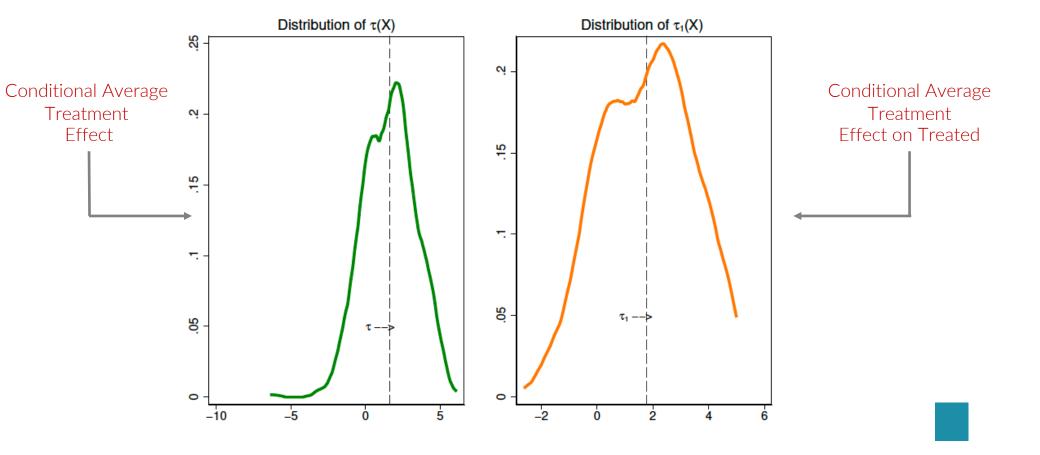
TARGET: Effect of a 1976 job training program on people real earnings in 1978

CONTROLS: age, race, educational attainment, previous employment condition, real earnings in 74 and 75



ESTIMATION OF ATE(X) AND ATET(X)

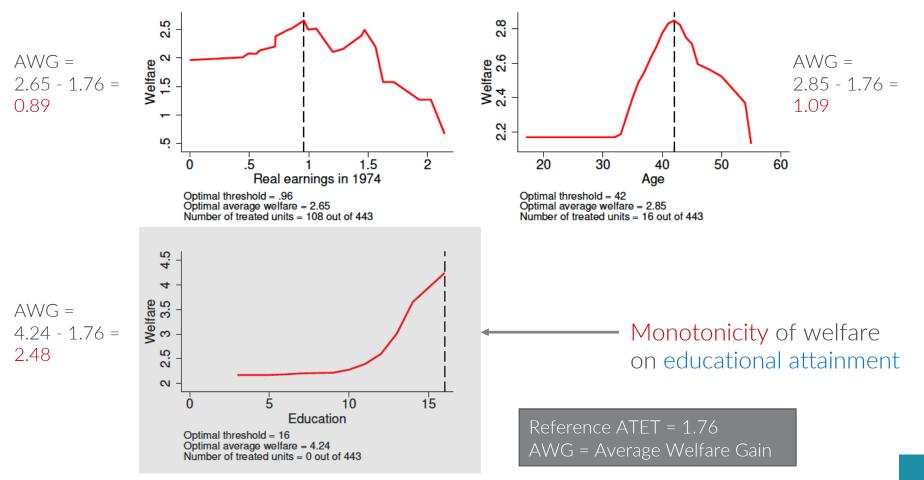
Figure 1: Distribution of $\hat{\tau}(X)$ and $\hat{\tau}_1(X)$. Program: National Supported Work Demonstration (NSWD). Data: LaLonde (1986). Target variable: Real earnings in 1978. Estimation technique: Regression-adjustment (with observable heterogeneity).





CONSTRAINED WELFARE MAXIMIZATION (UNIVARIATE)

Figure 2: Computation of the policy optimal selection threshold in univariate cases. Program: National Supported Work Demonstration (NSWD). Data: LaLonde (1986). Target variable: real earnings in 1978. Univariate selection variables: real earnings in 1974, age, and educational attainment.



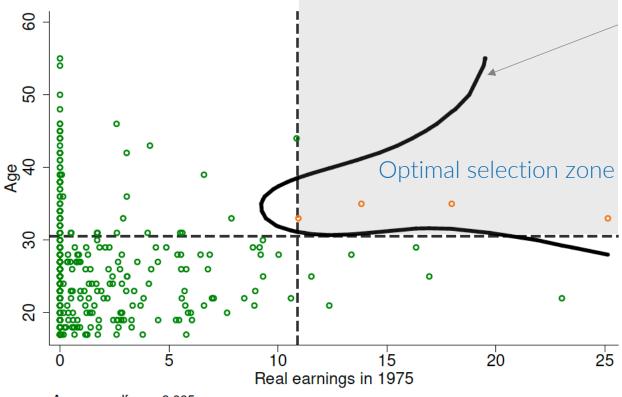


CONSTRAINED WELFARE MAXIMIZATION (BIVARIATE)

Figure 3: Computation of the policy optimal decision boundary in the bivariate case. Program: National Supported Work Demonstration (NSWD). Data: LaLonde (1986). Target variable: real earnings in 1978. Bivariate selection variables: real earnings in 1975 and age.

Reference ATET = 1.74

Average Welfare Gain = 3.99 - 1.74 = 2.255



Estimated Bayes optimal decision boundary

Average welfare = 3.995 Share of treated units = 1% Optimal threshold for 'Age' = 30.5 Optimal threshold for 'Real earnings in 1975' = 10.9



EMPIRICAL WELFARE MAXIMIZATION: RELEVANT ISSUES

1. Monotonicity

Welfare increases monotonically with a feature => too few to treat or too many to treat

2. Sparseness

X' comes from a different joint distribution than X

Trade-offs arising in this case, so the best to

do is offering the policymaker a "menu" of possible treatment choices given, for example, a pre-fixed budget

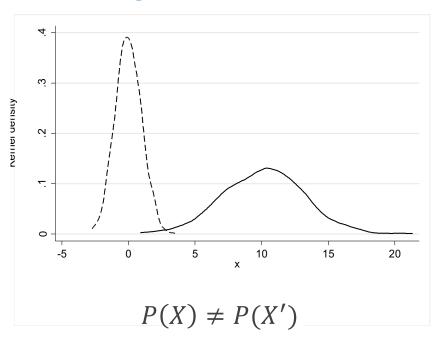




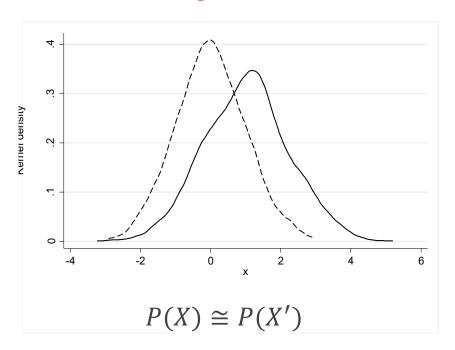
SPARSENESS

THE DISTRIBUTION OF X AND X' HAVE LOW OVERLAP

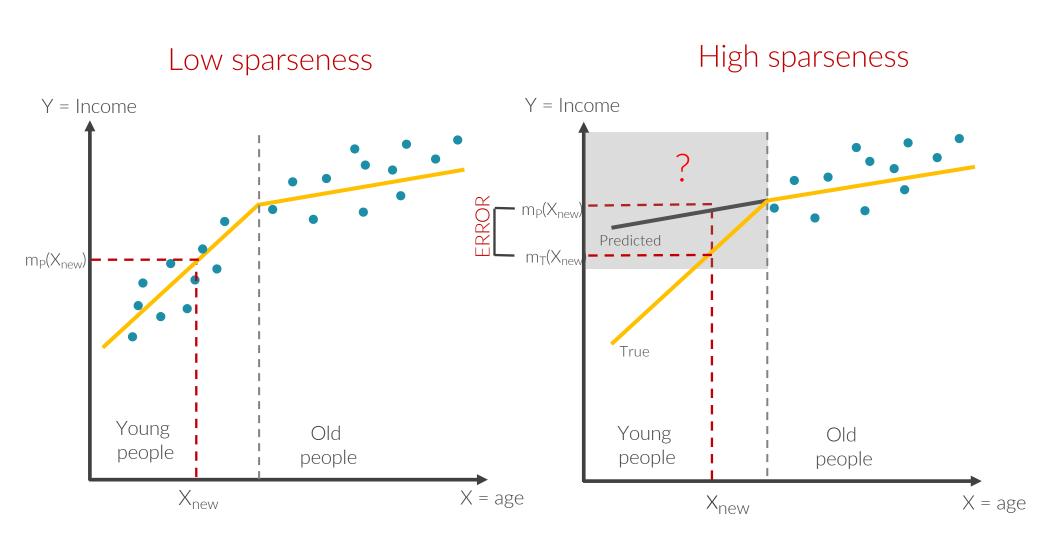
High sparseness



Low sparseness



Data sparseness weakens policy prediction





A SOLUTION TO MONOTONICITY TRADE-OFFS AND THE "MENU-STRATEGY"

FXAMPLE

Computation of policy optimal decision boundaries in the bivariate case, when one of the two selection variables (age) is fixed at its optimal threshold, and the threshold of the other variable (education) is varying. Program: National Supported Work Demonstration (NSWD). Data: LaLonde (1986). Target variable: real earnings in 1978. Bivariate selection variables: age and educational attainment.

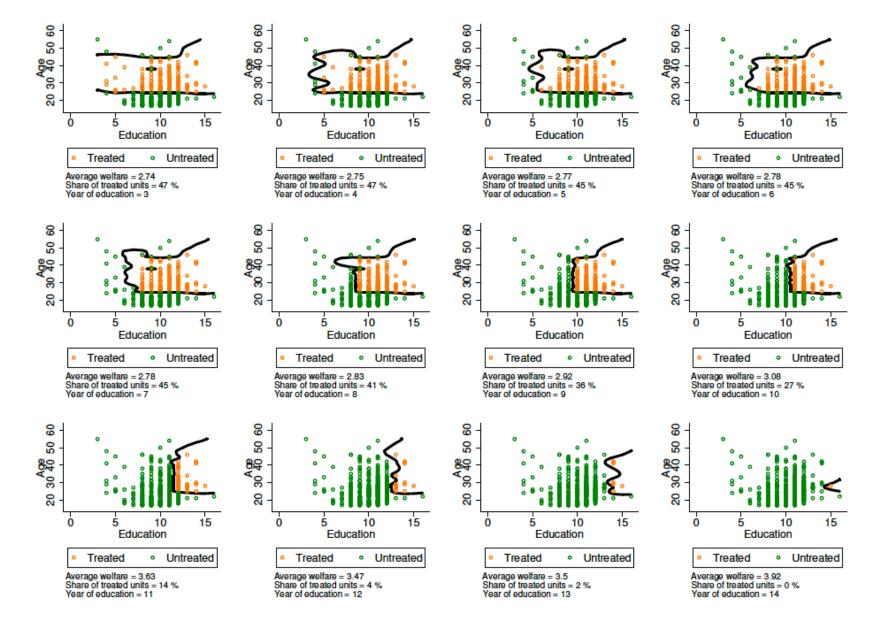
AGE -----> set at its optimal level

EDUCATION ----> free to vary

Feature plagued by monotonicity

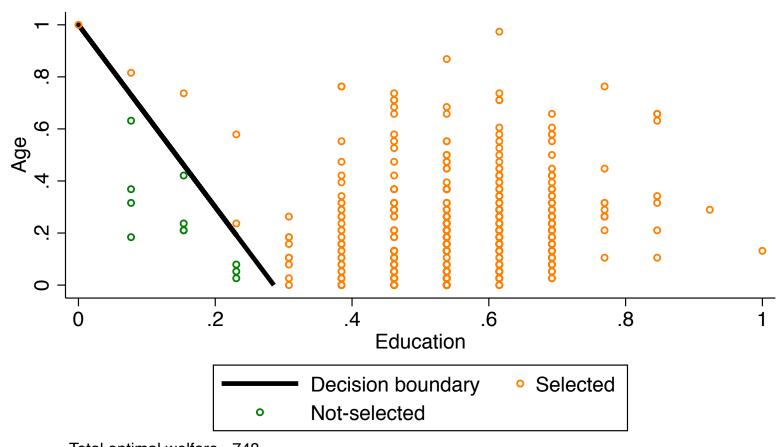


TRADE-OFFS AND THE "MENU-STRATEGY"





OPTIMAL SELECTION WITH A LINEAR COMBINATION POLICY



Total optimal welfare =748
Total oracle welfare = 764
Regret (absolute) = 15.53
Regret (%) = 2.03
Average welfare = 2.24
Average oracle welfare = 2.23
Share of treated units = 75 %





SOFTWARE

We formed a research group for OPL software implementation. We will develop a Policy Making Platform within the PNNR FOSSR project



Stata

Cerulli (CNR), opl package

R

Guardabascio (Perugia University) and Brogi (Istat)

Python

De Fausti (Istat)



THE STATA PACKAGE "OPL" (CERULLI 2023)

The commands of the Stata package OPL

```
Optimal policy learning with a threshold-based policy

opl_tb

Threshold-based optimal policy learning

opl_tb_c

Threshold-based policy learning at specific threshold values
```

Optimal policy learning with a linear-combination policy

```
opl_lc
    Linear-combination optimal policy learning
opl_lc_c
    Linear-combination policy learning at specific parameters' values
```

Optimal policy learning with a decision-tree policy

```
opl_dt
    Decision-tree optimal policy learning
Opl_dt_c
```

Decision-tree policy learning at specific splitting variables and threshold values





THRESHOLD-BASED POLICY

```
opl_tb — Threshold-based optimal policy learning

Syntax

opl_tb , xlist(var1 var2) cate(varname)

Description

opl_tb is a command implementing optimal ex-ante treatment assignment using as policy class a threshold-based (or quadrant) approach.
```

```
opl_tb_c —

Threshold-based policy learning at specific threshold values

Syntax

opl_tb_c , xlist(var1 var2) cate(varname) cl(number) c2(number) [graph]

Description

opl_tb_c is a command implementing ex-ante treatment assignment using as policy class a threshold-based (or quadrant) approach at specific threshold values c1 and c2 for respectively the selection variables var1 and var2.
```





LINEAR-COMBINATION POLICY

```
opl_lc — Linear-combination optimal policy learning

Syntax

    opl_lc , xlist(var1 var2) cate(varname)

Description

opl_lc is a command implementing optimal ex-ante treatment assignment using as policy class a linear-combination of variables var1 and var2: c1*var1+c2*var2=c3.
```

```
opl_lc_c —

Linear-combination policy learning at specific parameters' values

Syntax

opl_lc_c , xlist(var1 var2) cate(varname) cl(number) c2(number) [graph]

Description

opl_lc_c is a command implementing ex-ante treatment assignment using as policy class a linear-combination approach at specific parameters' values c1, c2, and c3 for the linear-combination of variables var1 and var2: c1*var1+c2*var2=c3.
```





DECISION-TREE POLICY

```
opl_dt — Decision-tree optimal policy learning

Syntax

    opl_dt , xlist(var1 var2) cate(varname)

    Description

opl_dt is a command implementing optimal ex-ante treatment assignment using as policy class a fixed-depth (1-layer) decision-tree based on selection variables var1 and var2.
```

```
opl_dt_c —

Decision—tree policy learning at specific splitting variables and threshold values

Syntax

opl_dt_c , xlist(var1 var2) cate(varname) c1(number) c2(number) [graph]

Opl_dt_c is a command implementing ex—ante treatment assignment using as policy class a fixed—depth (1-layer) decision—tree at specific splitting variables and threshold values.
```





THE "MAKE_CATE" COMMAND

make_cate —

Predicting conditional average treatment effect (CATE) on a new policy based on the training over an old policy

Syntax

make_cate outcome features , treatment(varname) model(model_type) new_cate(name) train_cate(name) new_data(name)

Description

make_cate is a command generating conditional average treatment effect (CATE) for both a training dataset and a testing (or new) dataset related to a binary (treated vs. untreated) policy program. It provides the main input for runni b opl_tb} (optimal policy learning of a threshold-based policy), opl_tb_c (optimal policy learning of a threshold-based policy at specific thresholds), opl_tc (optimal policy learning of a linear-combination policy at specific parameters), opl_dt (optimal policy learning of a decision-tree policy), opl_dt_c (optimal policy learning of a decision-tree policy at specific thresholds and select ables). Based on Kitagawa and Tetenov (2018), the main econometrics supported by these commands can be found in Cerulli (2022).





APPLICATION 1 - "OPL_TB_C"

```
Load initial dataset
    sysuse JTRAIN2, clear
Split the original data into a "old" (training) and "new" (testing) dataset
    qet train test, dataname(jtrain) split(0.60 0.40) split var(svar) rseed(101)
Use the "old" dataset (i.e. policy) for training
    use jtrain_train , clear
Set the outcome
    global v "re78"
Set the features
    global x "re74 re75 age agesg nodegree"
Set the treatment variable
    global w "train"
Set the selection variables
    global z "age mostrn"
Run "make_cate" and generate training (old policy) and testing (new policy) CATE predictions
    make_cate $y $x , treatment($w) model("ra") new_cate("my_cate_new") train_cate("my_cate_train") new_data("jtrain_test")
Generate a global macro containing the name of the variable "cate_new"
    global T `e(cate new)'
Select only the "new data"
    keep if _train_new_index=="new"
Drop "my_cate_train" as in the new dataset treatment assignment and outcome performance are unknown
    drop my_cate_train $w $y
Run "opl_tb" to find the optimal thresholds
    opl_tb , xlist($z) cate($T)
Save the optimal threshold values into two global macros
    global c1_opt=e(best_c1)
    global c2 opt=e(best c2)
Run "opl_tb_c" at optimal thresholds and generate the graph
    opl_tb_c , xlist($z) cate($T) c1($c1_opt) c2($c2_opt) graph
Tabulate the variable "_units_to_be_treated"
    tab _units_to_be_treated , mis
```



Policy class: Threshold-based

Main results

Learner = Regression adjustment
N. of units = 178
Threshold value c1 = .60000002
Average unconstrained welfare = 2.0673337
Percentage of treated = 1.1
N. of untreated = 176

Target variable =
Selection variables = age mostrn
Threshold value c2 = .79999999
Average constrained welfare = 2.885844
N. of treated = 2

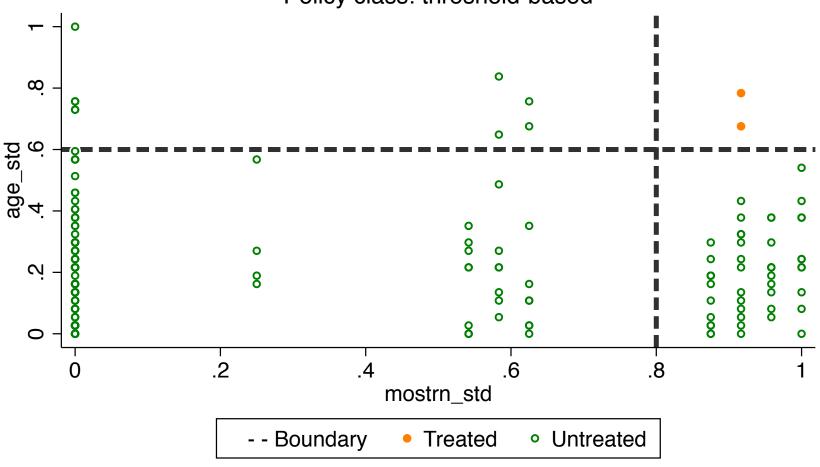
. tab _units_to_be_treated , mis

1 = unit to treat; 0 =			
unit not to treat	Freq.	Percent	Cum.
	11041	T CT CCITE	- Cum
0	176	98.88	98.88
1	2	1.12	100.00
Total	178	100.00	



Optimal policy assignment





Expected unconstrained average welfare = 2.07 Expected constrained average welfare = 2.89 Percentage of treated units = 1.1%







APPLICATION 2 - "OPL_LC_C"

```
Load initial dataset
    sysuse JTRAIN2, clear
Split the original data into a "old" (training) and "new" (testing) dataset
    qet train test, dataname(jtrain) split(0.60 0.40) split var(svar) rseed(101)
Use the "old" dataset (i.e. policy) for training
    use jtrain_train , clear
Set the outcome
    global v "re78"
Set the features
    global x "re74 re75 age agesq nodegree"
Set the treatment variable
    global w "train"
Set the selection variables
    global z "age mostrn"
Run "make_cate" and generate training (old policy) and testing (new policy) CATE predictions
    make cate $y $x , treatment($w) model("ra") new_cate("my_cate_new") train_cate("my_cate_train") new_data("jtrain_test")
Generate a global macro containing the name of the variable "cate new"
    global T `e(cate_new)'
Select only the "new data"
    keep if _train_new_index=="new"
Drop "my cate train" as in the new dataset treatment assignment and outcome performance are unknown
    drop my_cate_train $w $y
Run "opl_lc" to find the optimal linear-combination parameters
    opl lc , xlist($z) cate($T)
Save the optimal linear-combination parameters into three global macros
    global c1_opt=e(best_c1)
    global c2_opt=e(best_c2)
    global c3_opt=e(best_c3)
Run "opl_lc_c" at optimal linear-combination parameters and generate the graph
    opl_lc_c , xlist($z) cate($T) c1($c1_opt) c2($c2_opt) c3($c3_opt) graph
Tabulate the variable " units to be treated"
    tab _units_to_be_treated , mis
```



Policy class: Linear-combination

Main results

Learner = Regression adjustment

N. of units = 178

Lin. comb.parameter c1 = .59999999

Lin. comb.parameter c3 = .8

Average constrained welfare = 2.885844

N. of treated = 2

Target variable =
Selection variables = age mostrn
Lin. comb.parameter c2 = .45000001
Average unconstrained welfare = 2.0673337
Percentage of treated = 1.1
N. of untreated = 176

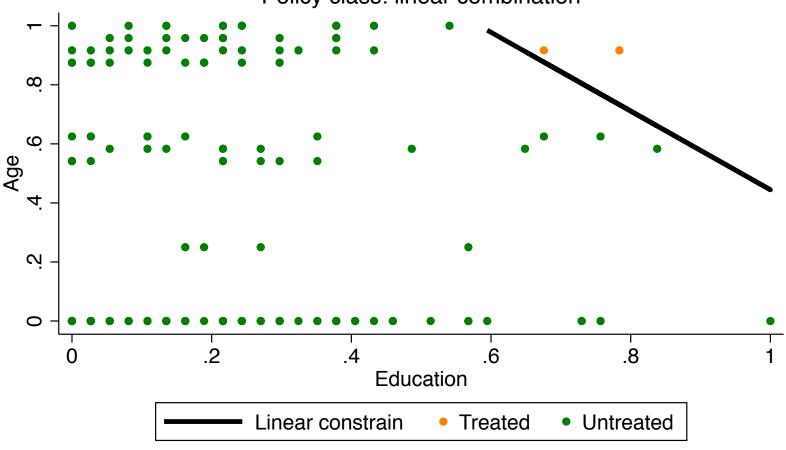
. tab _units_to_be_treated , mis

Total	178	100.00	
1	2	1.12	100.00
0	176	98.88	98.88
treat	Freq.	Percent	Cum.
unit not to			
treat; 0 =			
1 = unit to			



Optimal policy assignment





Expected unconstrained average welfare = 2.07 Expected constrained average welfare = 2.89 Percentage of treated units = 1.1%





APPLICATION 3 - "OPL_DT_C"

```
Load initial dataset
    sysuse JTRAIN2, clear
Split the original data into a "old" (training) and "new" (testing) dataset
   get train test, dataname(jtrain) split(0.60 0.40) split var(svar) rseed(101)
Use the "old" dataset (i.e. policy) for training
   use jtrain_train , clear
Set the outcome
   global y "re78"
Set the features
   global x "re74 re75 age agesq nodegree"
Set the treatment variable
   global w "train"
Set the selection variables
   global z "age mostrn"
Run "make_cate" and generate training (old policy) and testing (new policy) CATE predictions
   make_cate $y $x , treatment($w) model("ra") new_cate("my_cate_new") train_cate("my_cate_train") new_data("jtrain_test")
Generate a global macro containing the name of the variable "cate new"
   global T `e(cate_new)'
Select only the "new data"
    keep if _train_new_index=="new"
Drop "my_cate_train" as in the new dataset treatment assignment and outcome performance are unknown
    drop my_cate_train $w $y
Run "opl_dt" to find the optimal linear-combination parameters
   opl_dt , xlist($z) cate($T)
Save the optimal splitting variables into three global macros
   global x1_opt `e(best_x1)'
   global x2_opt `e(best_x2)'
   global x3_opt `e(best_x3)'
Save the optimal splitting thresholds into three global macros
   global c1_opt=e(best_c1)
   global c2_opt=e(best_c2)
   global c3_opt=e(best_c3)
Run "opl_dt_c" at optimal splitting variables and corresponding thresholds and generate the graph
   opl_dt_c , xlist($z) cate($T) c1($c1_opt) c2($c2_opt) c3($c3_opt) x1($x1_opt) x2($x2_opt) x3($x3_opt) graph
Tabulate the variable "_units_to_be_treated"
   tab _units_to_be_treated , mis
```



Policy class: Fixed-depth decision-tree

Main results

Learner = Regression adjustment
N. of units = 178
Threshold first splitting var. = .69999999
Threshold third splitting var. = = .60000002
Average constrained welfare = 4.2417823
N. of treated = 3
First splitting variable x1 = age
Third splitting variable x3 = age

Target variable =
Selection variables =
Threshold second splitting var. = .89999998
Average unconstrained welfare = 2.0673337
Percentage of treated = 1.7
N. of untreated = 175
Second splitting variable x2 = age

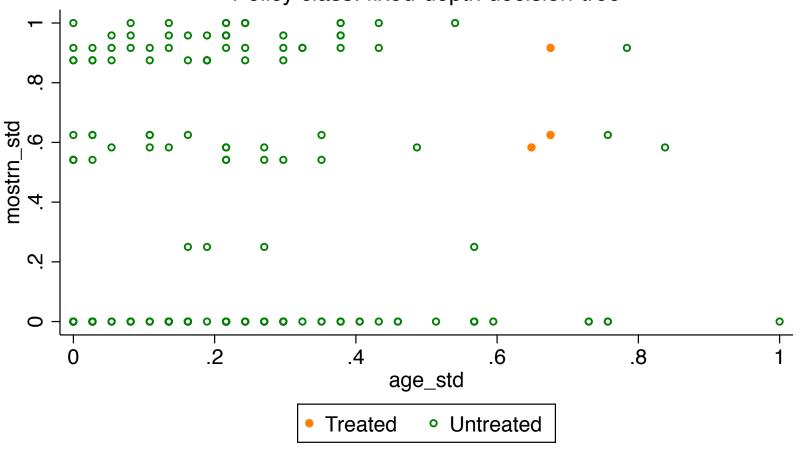
. tab _units_to_be_treated , mis

Total	178	100.00	
1	3	1.69	100.00
0	175	98.31	98.31
treat	Freq.	Percent	Cum.
unit not to			
treat; 0 =			
1 = unit to			



Optimal policy assignment

Policy class: fixed-depth decision-tree



Expected unconstrained average welfare = 2.07 Expected constrained average welfare = 4.24 Percentage of treated units = 1.7%



CONCLUSIONS AND FUTURE AVENUES

- □ Policy Learning: new frontier of econometrics of prog evaluation
- ☐ Theory-driven and data-driven approaches can complement
- Extensions to unobservable selection quite straightforward
- Welfare monotonicity and data sparseness major problems
- Monotonicity solved by "menu strategy"
- ☐ Generalization to other policy classes
- □ OPL with multiple treatments
- □ OPL with continuous treatments



Thanks for your kind attention!