Multi-Dimensional
Well-Being, Deprivation, and Inequality

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Outline

[1] Framing – Multidimensional Measures for WellBeing Applications

[2] MPI | MWI – Deprivation & Well-Being | **AF (IR)** | Fixed-Fuzzy-Aproach

[3] M|K|GG – Inequality (vertikal|horizontal) | **Gini |** Subgroup-Indices

[4] Programs – MPI | MWI | MGDIV || DimWeight | Rescale

[5] Outlook | Discussion

Framing

[1] **Framing – Why Multi-dimensional WellBeing Applications?**

1.1 Identifying individual overlaps in well-being indicators {across conception levels}

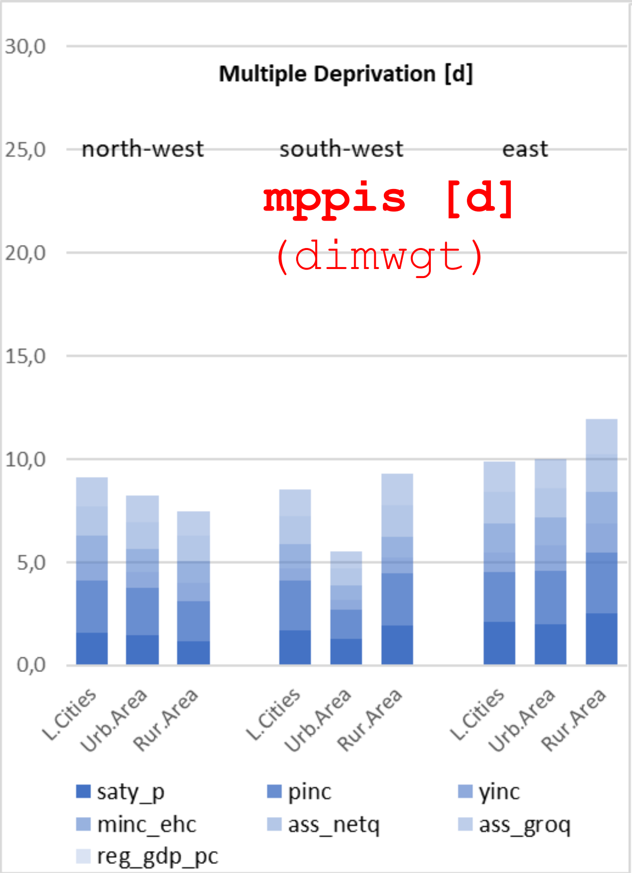
1.2 Identifying shifts in multiple deprivations | well-beings | inequalities {for opinion dynamics}

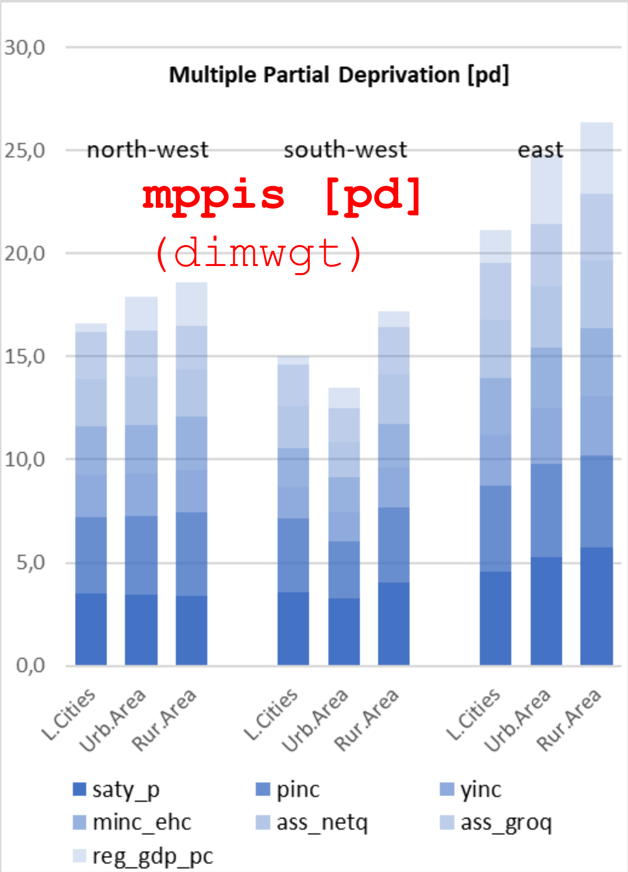
1.3 Providing structured (output) indicators for statistical (regression) analyses {individual scores}

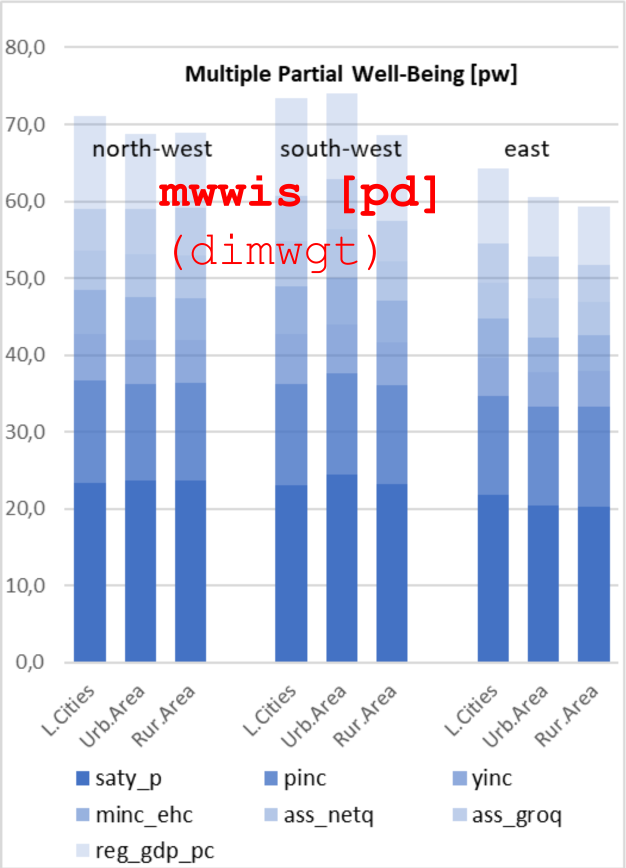
Programs	**mppis**	**// multi-dimensional Deprivation and Poverty**
	mwwis	**// multi-dimensional WellBeing and Wealth**
	mgdiv	**// multi-dimensional Inequality | Gini**
Tools	**dimwgt**	**// multi-dimensional weighting**
	rescale	**// multi-dimensional rescale**

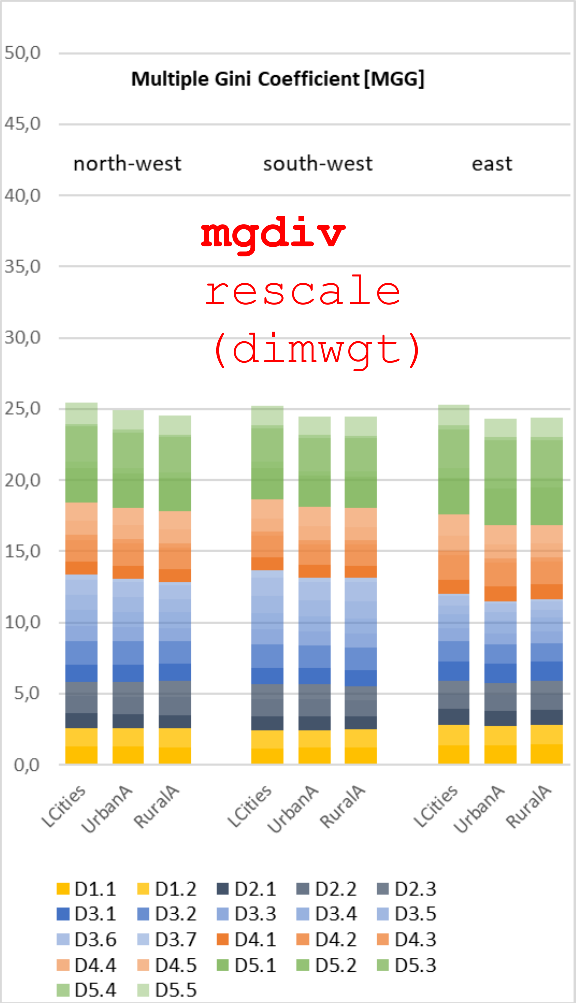
All programs provide -new- features to adress issues of deprivation, wealth and inequalities not only as tensions between rich and poor - but also as variations of wellbeing for the entire majority of population with more ordinary living standards.

M-Dim – Deprivation & Well-Being [standard and fixed-fuzzy approach, AF] | Inequalities [M-Gini]









SOEPv37 [2017; total population | adults 20+]

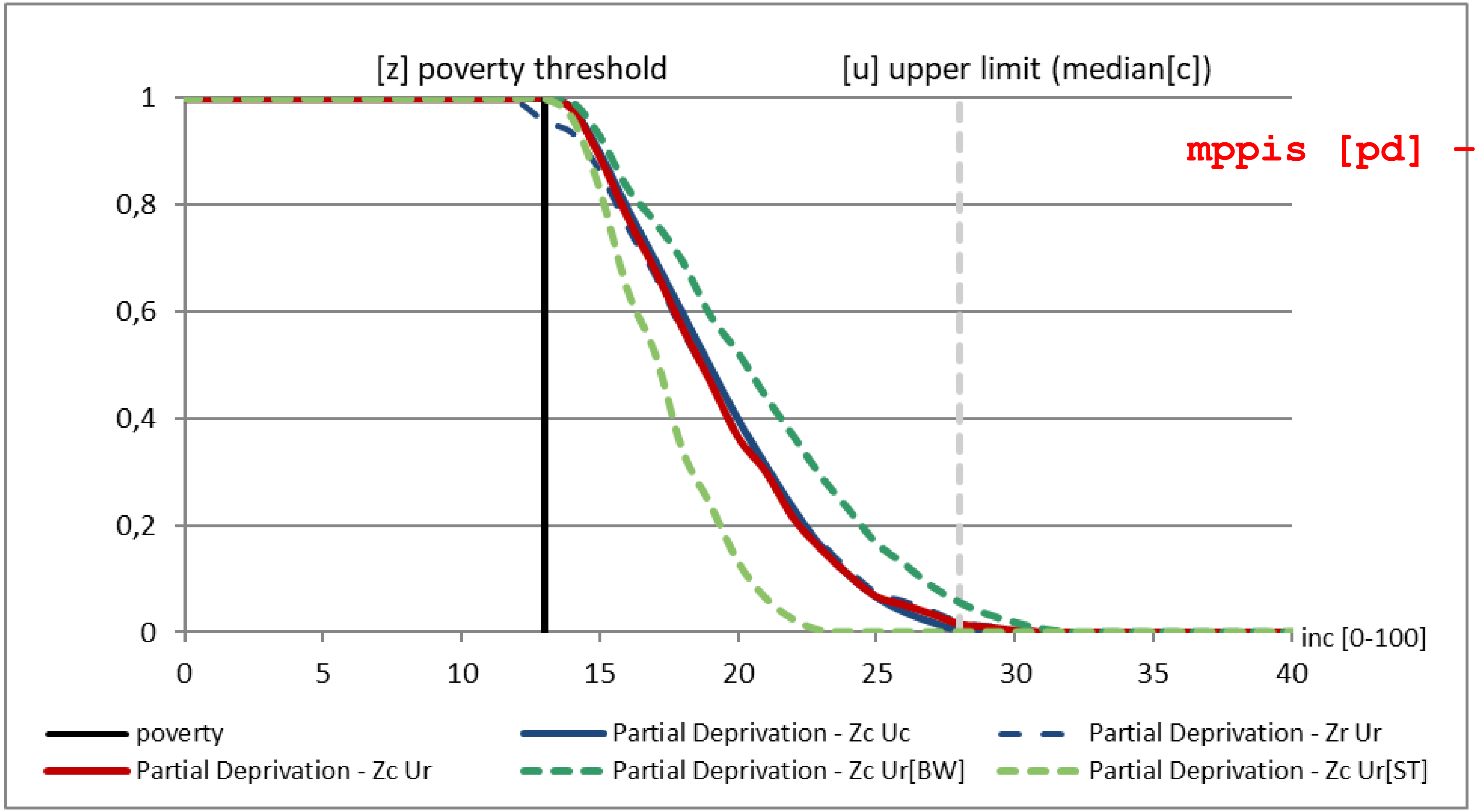
Krause (2024) [forthcoming]

Levels	*Design*	*Dwgt(min)*	*Dwgt(max)*	*SubDimensioı*	Indicators	Variables
regional	0.2	.131	.185	reg.income	reg_gdp_pc	Reg.GDP per capita
social [ass.]	0.1	.080	.107	hh.asset	ass_groq	Gross-HH.Assets
	0.1	.079	.106	hh.asset	ass_netq	Net-HH.Assets [>=0]
social [hinc.]	0.1	.098	.128	hh.income	minc_ehc	Monthly eq.Hh-Income EHC
	0.1	.099	.140	hh.income	yinc	Annual eq.Hh-Income
personal	0.2	.164	.231	ind.income	pinc	Personal Annual Income
intra-persona	0.2	.211	.267	sat.income	saty_p	Sat.Hhold-Income

SOEPv37. Design: dimensional design [Sum=1]; Dwgt: dimensional weights [Sum=1];
Household-Income EHC: Household Income after eligible housing costs.

	Dimension	SubDimension
D1	Life	health
D2	Home	housing
D3	Econ.Resources	income, assets
D4	Soc.Resources	qual.of.life, participation
D5	Cult.Resources	work,skills,handicap

Degrees of (partial) deprivation in national and regional settings (Germany, 2018)



mppis [pd] – one-dimensional

z_c national pov-line [country]
z_r regional pov-line [region]

u_c national upper-line [country]
u_r regional upper-line [region]

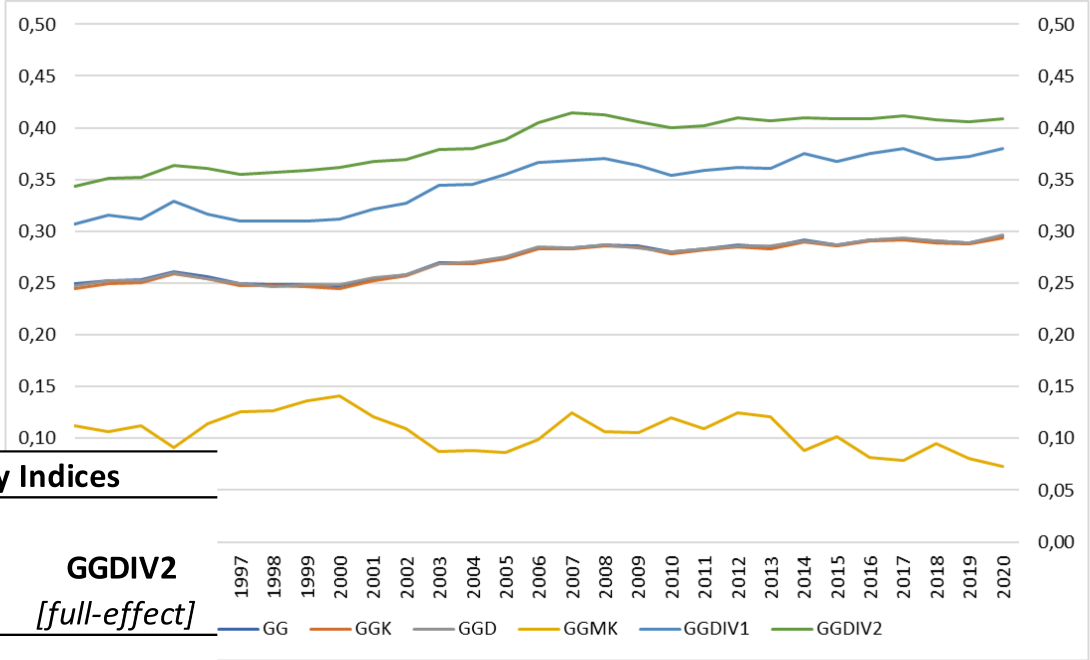
Baden-Wuerttemberg [BW] and Saxony-Anhalt [ST] [SOEPv35, 2018: Med-C: 22.431 EUR; Med-BW: 24.982 EUR; Med-ST: 18.318 EUR].

Results refer to the entire national population [C].

Inequality and Diversity

HH-Net-Eq.Incomes

mgdiv – one-dimensional



2020 Coeff. {0,..,1}		Gini Sub-Indices GG [main]	GGK [within-Gr.]	GGD [between-Gr.]	GGMK [Gr-diff]	Diversity Indices GGDIV1 [net-effect]	GGDIV2 [full-effect]
Total		*0,295*	*0,294*	*0,296*	*0,073*	*0,380*	*0,409*
N-West	*(Total)*	*0,314*	*0,313*	*0,314*	*0,034*	*0,418*	*0,432*
	LargeCities	0,339	0,360	0,334	0,052	0,437	0,458
	UrbanArea	0,314	0,314	0,314	0,031	0,419	0,432
	RuralArea	0,292	0,270	0,299	0,028	0,400	0,412
S-West	*(Total)*	*0,285*	*0,283*	*0,289*	*0,101*	*0,362*	*0,401*
	LargeCities	0,349	0,403	0,342	0,133	0,439	0,485
	UrbanArea	0,283	0,280	0,287	0,094	0,361	0,398
	RuralArea	0,267	0,247	0,276	0,105	0,337	0,378
East	*(Total)*	*0,267*	*0,266*	*0,273*	*0,084*	*0,348*	*0,380*
	LargeCities	0,291	0,311	0,283	0,119	0,353	0,398
	UrbanArea	0,262	0,256	0,263	0,051	0,341	0,363
	RuralArea	0,258	0,247	0,271	0,077	0,348	0,377

SOEPv37. Med.Annual.Household-Net-Equivalent Incomes [rev.OECD-scale; €-2019, previous year];
GG… population adjusted Gini Sub-Indices.

yinc	D1	GGD Gini Sub-Indices [between-G]			D1	KGGD Gini Sub-Indices [between-G]		
ACReg	H1	**2000**	**2010**	**2020**	H1/4	**2000**	**2010**	**2020**
Total		*0,249*	*0,280*	*0,293*		*0,250*	*0,281*	*0,295*
N-West	BigCity	*0,260*	*0,309*	*0,325*		*0,263*	*0,314*	*0,330*
N-West	City|UrbanArea	*0,252*	*0,288*	*0,299*		*0,255*	*0,292*	*0,302*
N-West	RuralArea	*0,257*	*0,260*	*0,276*		*0,259*	*0,260*	*0,276*
S-West	BigCity	*0,287*	*0,315*	*0,351*		*0,290*	*0,317*	*0,356*
S-West	City|UrbanArea	*0,247*	*0,285*	*0,293*		*0,248*	*0,288*	*0,295*
S-West	RuralArea	*0,238*	*0,264*	*0,282*		*0,239*	*0,263*	*0,282*
East	BigCity	*0,271*	*0,276*	*0,287*		*0,273*	*0,277*	*0,289*
East	City|UrbanArea	*0,222*	*0,243*	*0,256*		*0,222*	*0,243*	*0,256*
East	RuralArea	*0,223*	*0,253*	*0,264*		*0,219*	*0,249*	*0,261*

SOEPv38.1. Total population. D1: [yinc] hh-net-equivalent income, at prices of 2021. Goups H1: [acreg {9}] area-city-region; H2: [ager{6}] age-Groups; H3: [smig{6}] sex-migration; H4: [emp5{5}] employment-status.
KGGD - empirical individual dimensional weights (d1|h1/4) derived from hpsatl (life satisfaction, adj. for.tot.pop).
KGGD - Gini indices -adjusted for between-group population [GG..] and population of subgroup levels.

One- and multi-dimensional between-group inequalities in regional settings

mgdiv – m|k-dimensional

D1/6	D1/6	MGGD Gini Sub-Indices [between-G]			D1/6	MKGGD Gini Sub-Indices [between-G]		
ACReg	H1	**2000**	**2010**	**2020**	H1/4	**2000**	**2010**	**2020**
Total		*0,196*	*0,196*	*0,174*		*0,198*	*0,197*	*0,175*
N-West	BigCity	*0,201*	*0,200*	*0,179*		*0,204*	*0,202*	*0,180*
N-West	City|UrbanArea	*0,193*	*0,194*	*0,174*		*0,195*	*0,195*	*0,175*
N-West	RuralArea	*0,190*	*0,191*	*0,170*		*0,194*	*0,192*	*0,173*
S-West	BigCity	*0,222*	*0,217*	*0,189*		*0,226*	*0,220*	*0,192*
S-West	City|UrbanArea	*0,197*	*0,196*	*0,172*		*0,199*	*0,197*	*0,175*
S-West	RuralArea	*0,189*	*0,193*	*0,170*		*0,192*	*0,194*	*0,173*
East	BigCity	*0,193*	*0,188*	*0,173*		*0,195*	*0,194*	*0,175*
East	City|UrbanArea	*0,193*	*0,186*	*0,167*		*0,196*	*0,191*	*0,170*
East	RuralArea	*0,196*	*0,196*	*0,173*		*0,195*	*0,196*	*0,171*

SOEPv38.1. Adult population (20+). D1/6: rescaled indices [vct1{1,...100}] of d1/d6 [yinc,minc,rgdpr,sat-hinc,sat-home,sat-heal].
Groups H1: [acreg {9}] area-city-region; H2: [ager{6}] age-Groups; H3: [smig{6}] sex-migration; H4: [emp5{5}] (n)empl.
M|KGGD - empirical individual dimensional weights (d1/6|h1,h1/4) derived from ln-hhincome,sat-life;
M|KGGD - Gini indices -adjusted for between-group population [GG..] and population of subgroup levels.

MPI | MWI

[2] Multidimensional Poverty Index [MPI] | Multidimensional WellBeing Index [MWI]

2.1 Fixed-Fuzzy-Approach – conception | absolute-relative | (integration, segmentation)

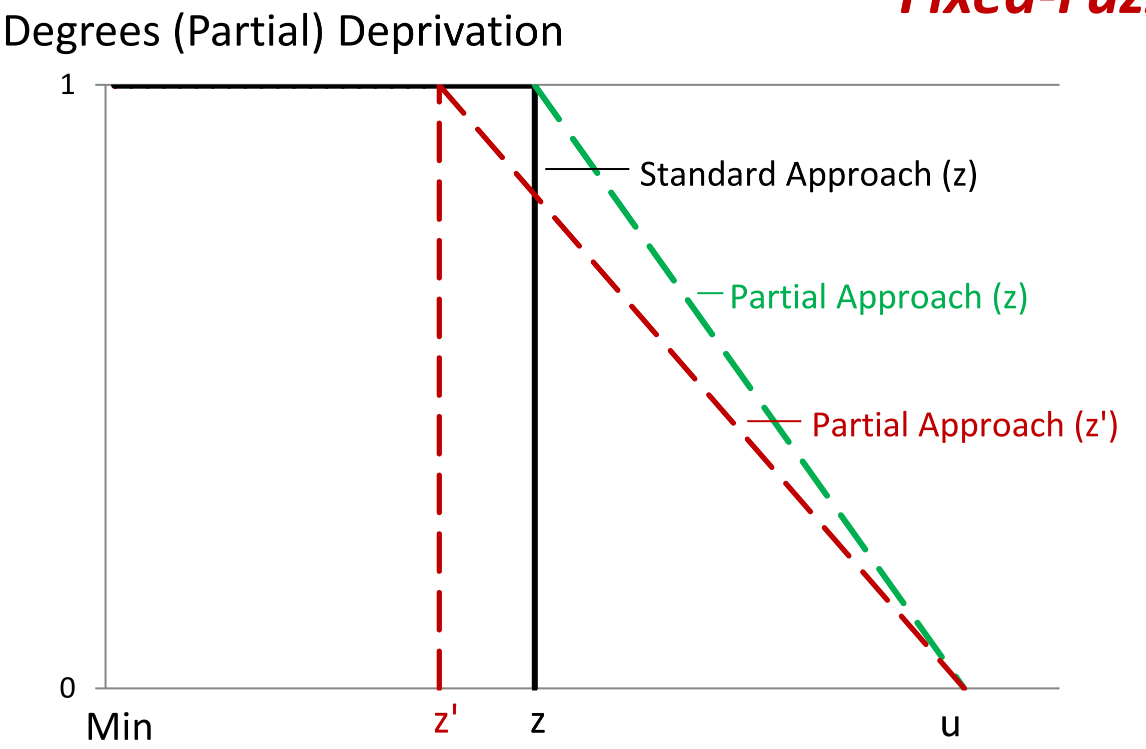
2.2 Identification - **AF** (dual-cutoff) {censored deprivation account} | also applied to WellBeing

2.3 Aggregation – (M0,M{α}) | FGT-Modification | non-linear cardinal assignments)

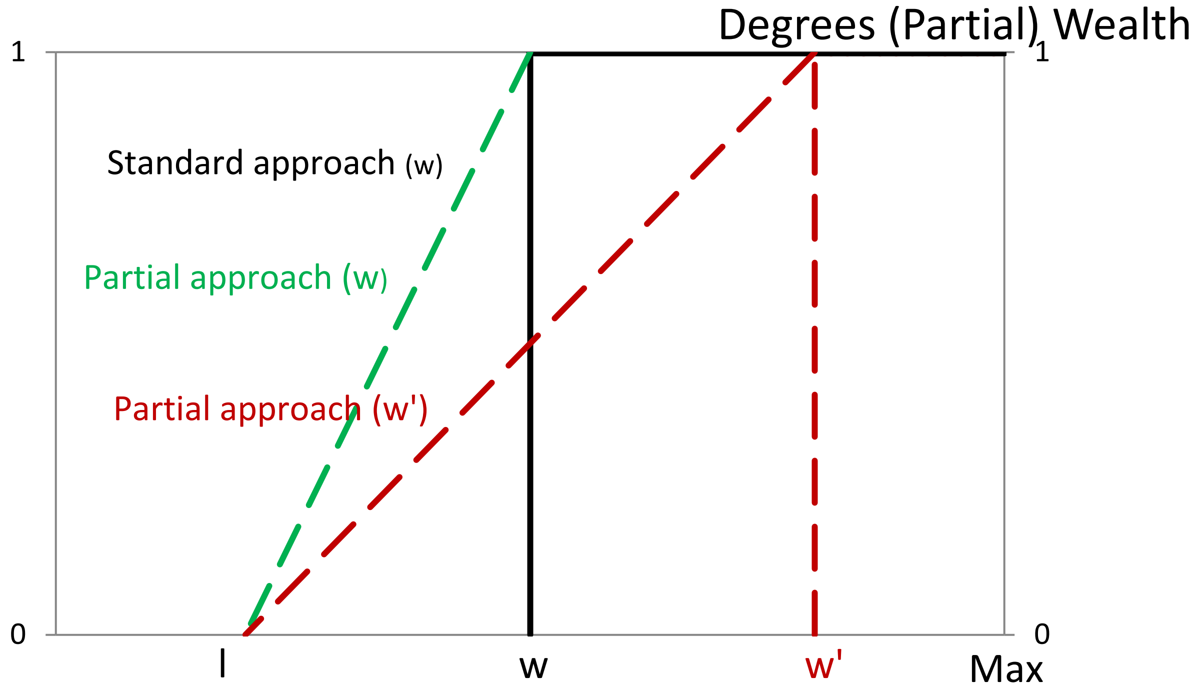
Fuzzy Zones of Precarity

Fuzzy Zones of Prosperity

Fixed-Fuzzy-Approach



- select indicator(s) [dimension]
- define fuzzy zones of prosperity [w, l]
- extract degrees of wealth
- run counting approach



- select indicator(s) [dimension]
- define fuzzy zones of precarity [z, u]
- extract degrees of deprivation
- run counting approach

[Krause (2019; 2024) forthcoming]

Identification of Deprivation [Poverty]

-- lack of capabilities --

$$hz_i = 1 \text{ if } y_i < z$$

$$hz_i = 0 \text{ if } y_i \geq z$$

with

y_i individual indicator for poverty
z poverty threshold
hz_i individual poverty score [0, 1]

Aggregation of Deprivation [Poverty]

- FGT0, 1, … {α} [adjusted FGT]
- M0, 1, … {α} [AF, dual-cutoff, .33]

Foster/Greer/Thorbecke 1984, 2010;
Alkire/Foster 2011a,b; Alkire et al. 2015

Identification of Well-Being [Wealth]

-- capabilities --

$$hw_i = 1 \text{ if } x_i > w$$

$$hw_i = 0 \text{ if } x_i \leq w$$

with

x_i individual indicator for wealth
w wealth threshold
hw_i individual wealth score [0, 1]

Aggregation of Well-Being [Wealth]

- FGT0, 1, … {α} [adjusted FGT]
- W0, 1, … {α} [AF, dual-cutoff, .33]

Peichl/Pestel 2013; Krause 2019;

Identification of Partial Deprivation

Identification of Partial Well-Being

Fixed-Fuzzy-Approach

$$pd_i = exp\left\{\varepsilon\left[1-\left(\frac{y_i}{z}\right)^{\tau\left[\left(\frac{y_i}{z}\right)\right]}\right]\right\}$$

with

pd_i individual score of partial deprivation for indicator y_i [0, …, 1]

y_i individual value of deprivation in indicator y

z threshold of poverty/deprivation for indicator y

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

$$pw_i = exp\left\{\varepsilon\left[1-\left(\frac{x_i}{w}\right)^{-\tau\left[\left(\frac{x_i}{w}\right)^{-1}\right]}\right]\right\}$$

with

pw_i individual score of partial well-being for indicator x_i [0, …, 1]

x_i individual value of well-being in indicator x

w threshold of well-being for indicator x

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

Aggregation of Partial Deprivation

- FGT0, 1, … {α} [adjusted FGT]
- M0, 1, … {α} [AF, dual-cutoff, .33]

Aggregation of Partial Well-Being

- FGT0, 1, … {α} [adjusted FGT]
- W0, 1, … {α} [AF, dual-cutoff, .33]

Krause 2019; 2024/forthcoming

[2] Identification of Partial Deprivation

$$pd_i = exp\left\{\varepsilon\left[1-\left(\frac{y_i}{z}\right)^{\tau\left[\left(\frac{y_i}{z}\right)\right]}\right]\right\}$$

with

pd_i individual score of partial deprivation for indicator y_i

y_i individual value of deprivation in indicator y

z threshold of poverty/deprivation for indicator y

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

[2] Identification of Partial Wealth

$$pw_i = exp\left\{\varepsilon\left[1-\left(\frac{x_i}{w}\right)^{-\tau\left[\left(\frac{x_i}{w}\right)^{-1}\right]}\right]\right\}$$

with

pw_i individual score of partial wealth for indicator x_i

x_i individual value of wealth in indicator x

w threshold of wealth for indicator x

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

[2] Identification of Partial Deprivation

Defining the "zone of precarity" (b)

$$b = \frac{u}{z}$$

with

z poverty/deprivation threshold
u upper limit for partial deprivation

[2] Identification of Partial Wealth

Defining the "zone of prosperity" (q)

$$q = \frac{l}{w}$$

with

w wealth threshold
l lower limit for partial wealth

[2] Identification of Partial Deprivation

Defining [lim] for the "zone of precarity" (b)

$$\lim\_u = exp\{\varepsilon[1 - b^{\tau[b]}]\}$$

with

$\lim\_u$ marginal rate of partial deprivation at the upper limit (u) [default: lim_u = .01]

b ratio of lower to upper limits (u / z) for the fuzzy zone of deprivation

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function [default: ε = 1.0]

[2] Identification of Partial Wealth

Defining [lim] for the "zone of prosperity" (q)

$$\lim\_l = exp\{\varepsilon[1 - q^{-\tau[q^{-1}]}]\}$$

with

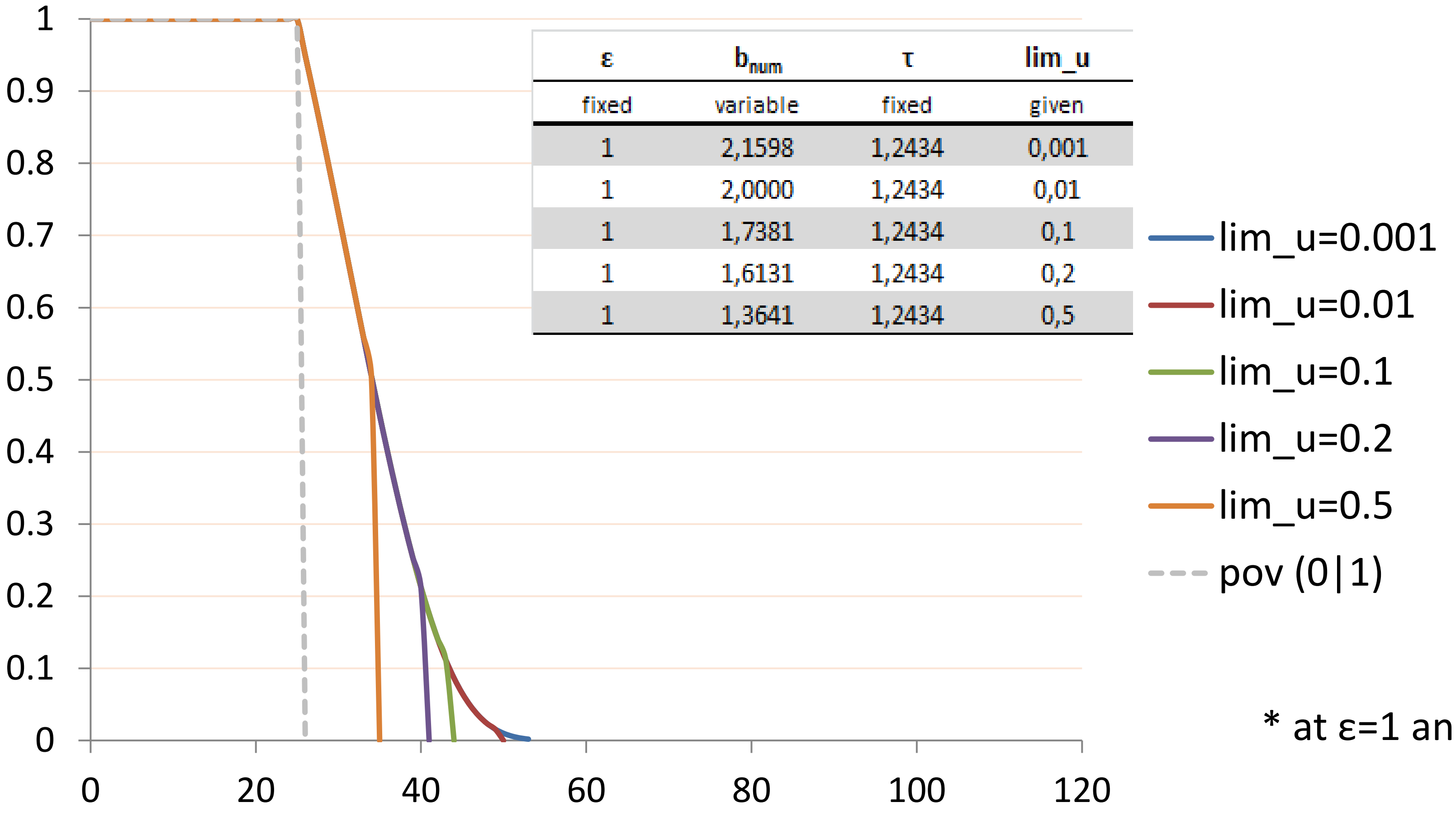
$\lim\_l$ marginal rate of partial wealth at the lower limit (l) [default: lim_ l = .01]

q ratio of lower to upper limits (l / w) for the fuzzy zone of prosperity

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function [default: ε = 1.0]

Different lim_u* → Segmentation Approach



ε	b_{num}	τ	lim_u
fixed	variable	fixed	given
1	2,1598	1,2434	0,001
1	2,0000	1,2434	0,01
1	1,7381	1,2434	0,1
1	1,6131	1,2434	0,2
1	1,3641	1,2434	0,5

* at ε=1 and τ=1,2434

[2] Identification of Partial Deprivation

Setting the baseline function (τ) for partial deprivation at the "zone of precarity":

$$\tau = -\frac{\ln\left[1 - \frac{\ln(\lim\_u)}{\varepsilon}\right]}{b\,\ln(b)}$$

Modelling the shape of the slope (ε) for the baseline function:

$$\varepsilon = \frac{\ln(\lim\_u)}{1 - b^{\tau[b]}}$$

[2] Identification of Partial Wealth

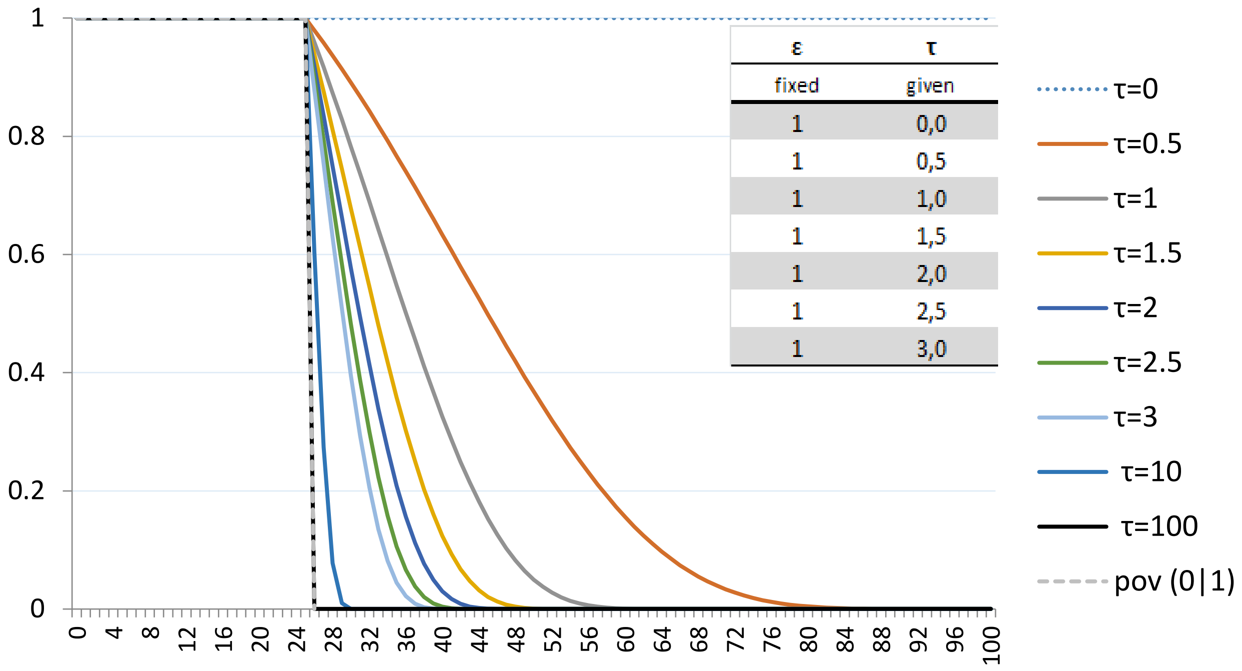
Setting the baseline function (τ) for partial wealth at the "zone of prosperity":

$$\tau = -\frac{\ln\left[1 - \frac{\ln(\lim\_l)}{\varepsilon}\right]}{q^{-1}\ln(q)}$$

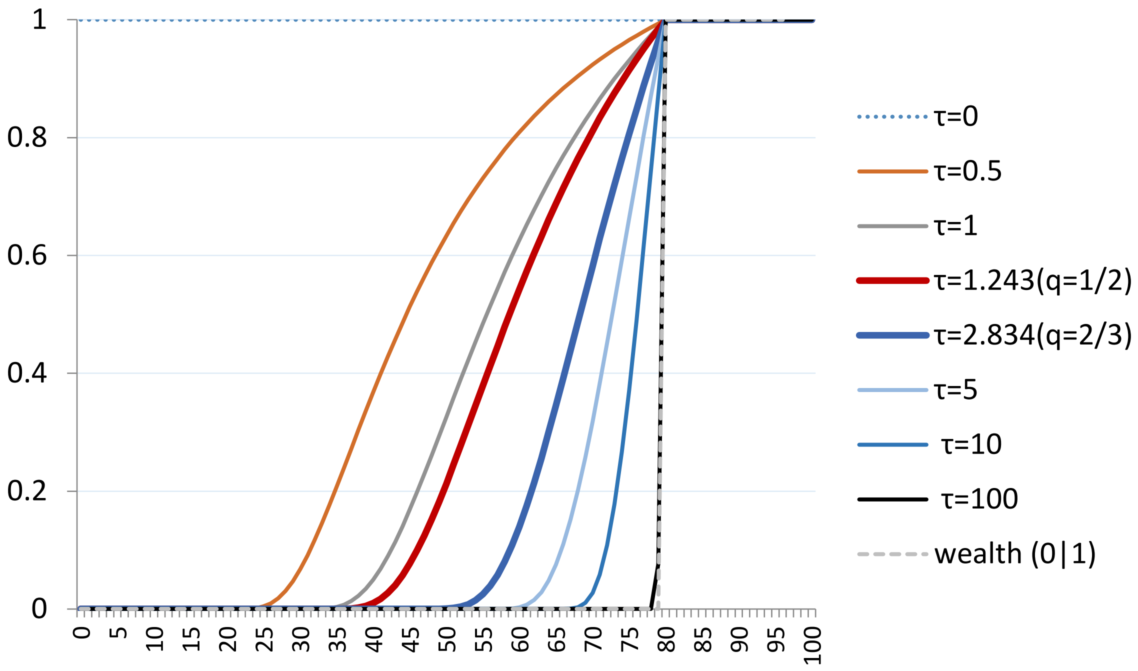
Modelling the shape of the slope (ε) for the baseline function:

$$\varepsilon = \frac{\ln(\lim\_l)}{1 - q^{-\tau[q^{-1}]}}$$

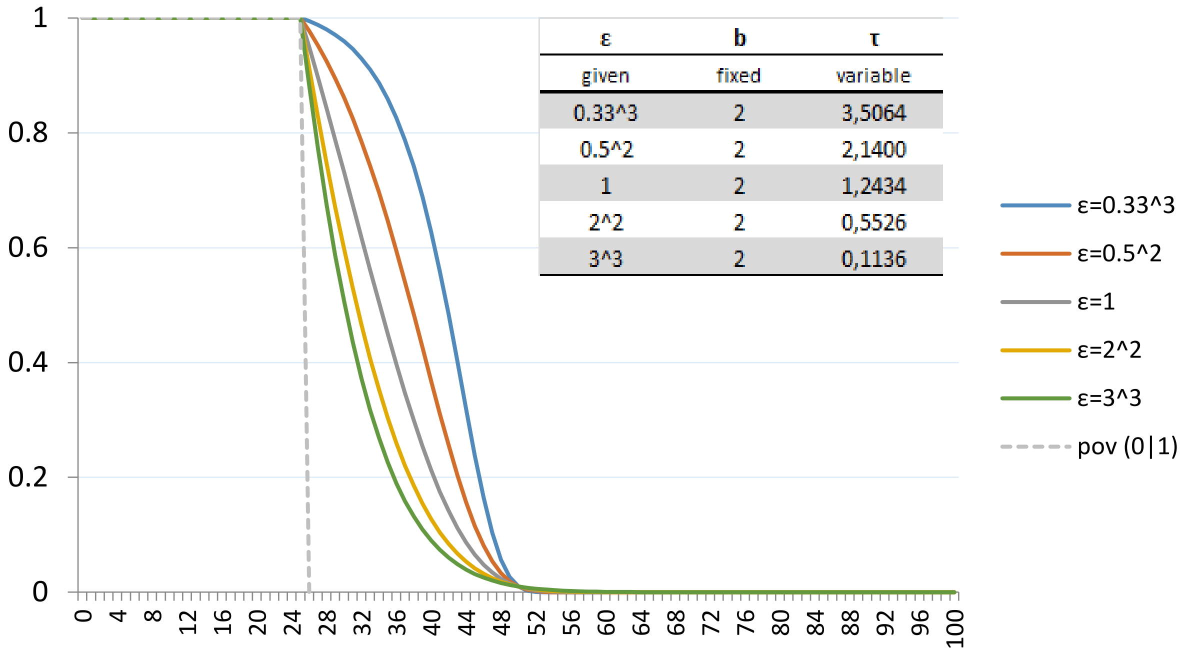
[3a] Identification of Poverty *and* Partial Deprivation *baseline function (τ)*



[3a] Identification of Wealth *and* Partial Wealth *baseline function (τ)*

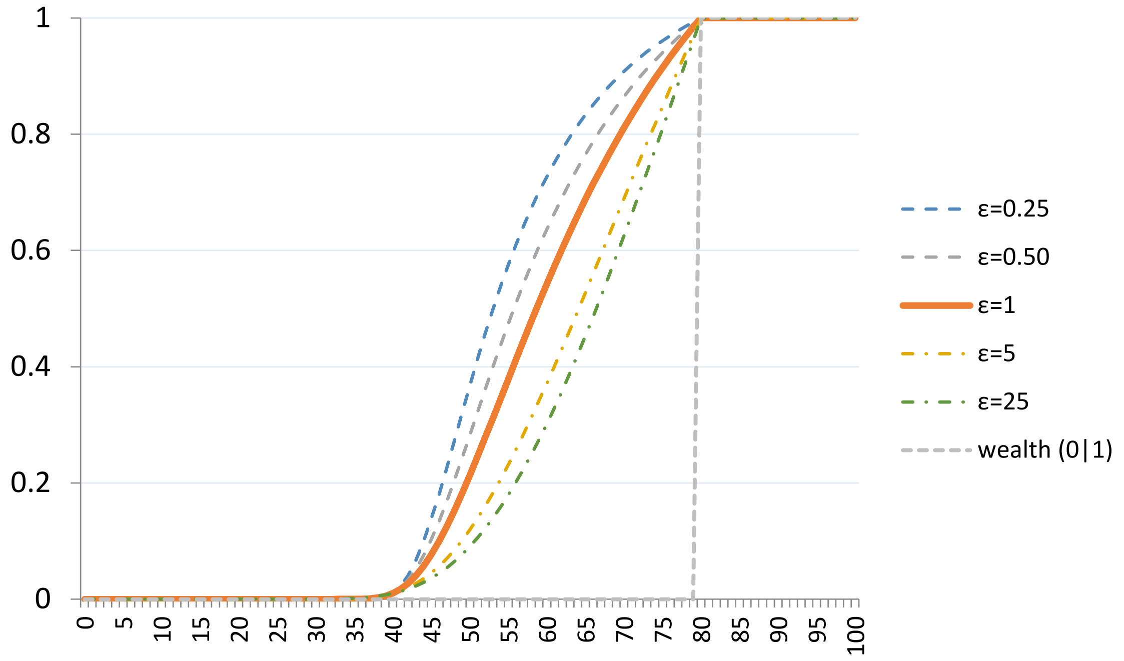


[3b] Identification of Poverty *and* Partial Deprivation <u>*shape of the slope (ε)*</u>



ε	b	τ
given	fixed	variable
0.33^3	2	3,5064
0.5^2	2	2,1400
1	2	1,2434
2^2	2	0,5526
3^3	2	0,1136

[3b] Identification of Wealth *and* Partial Wealth <u>*shape of the slope (ε)*</u>



[4] FGT-Measurement of Poverty

In 1984 Foster, Greer, and Thorbecke presented their now well-established FGT measures of poverty:

$$FGT = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{z - y_i}{z}\right)^{\alpha}$$

[4a] Modified FGT-Measurement of Poverty

for specifications of gaps:

$$FGTmod = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{z - y_i}{z - dmin}\right)^{\alpha}$$

[6] Adjusted FGT Aggregation of Poverty

$$y_i{}^* = \frac{z - y_i}{z - d_{min}} \qquad y_i{}^* \in [0, \dots, 1]$$

$$hz_i = \{0,1\} \quad \text{identification: } hz_i = 1 \text{ for } y_i < z$$

$$iz_i = iz(z)_i = \left(\frac{z - y_i}{z - d_{min}}\right)^{\alpha} \text{ for } y_i \le z$$

$$zFGT = \frac{1}{n} \sum_{j=1}^{n} hz_i \, iz_i$$

[6] Adjusted FGT Aggregation of Wealth

$$x_i{}^* = \frac{x_i - \mathrm{w}}{w_{max} - \mathrm{w}} \qquad x_i{}^* \in [0, \dots, 1]$$

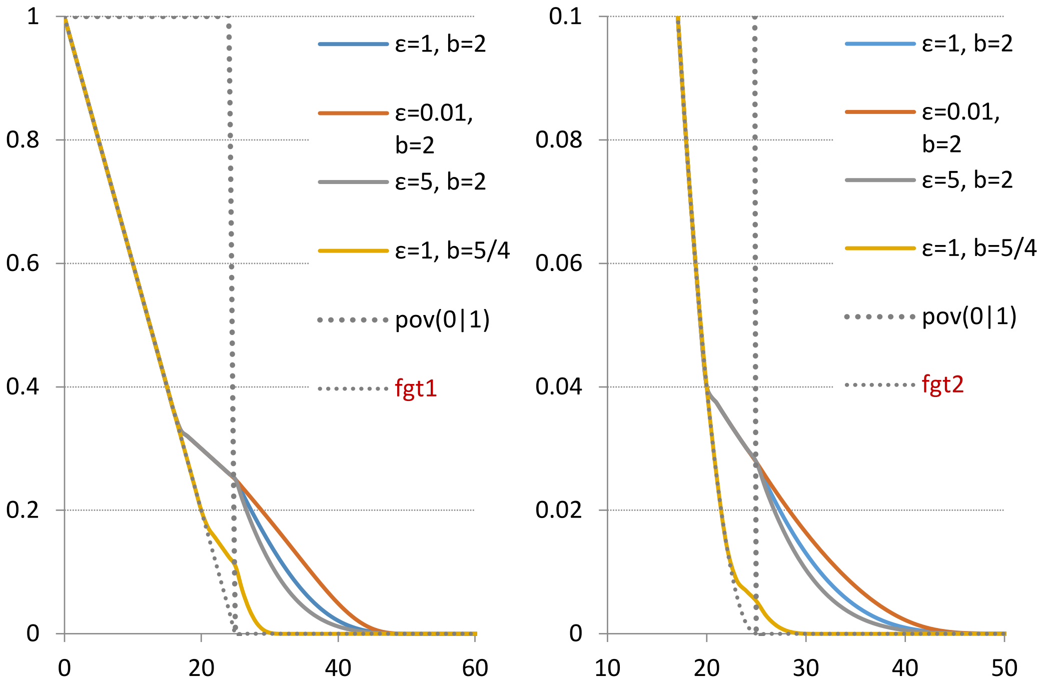
The proposed FGT measure for wealth can such be formulated for standard linear applications, similar to the corresponding poverty measure.

$$hw_i = \{0,1\} \quad \text{identification: } hw_i = 1 \text{ for } x_i > w$$

$$iw_i = iw(w)_i = \left(\frac{x_i - \mathrm{w}}{w_{max} - \mathrm{w}}\right)^{\alpha} \text{ for } x_i \ge w$$

$$wFGT = \frac{1}{n} \sum_{r=1}^{n} hw_i \, iw_i$$

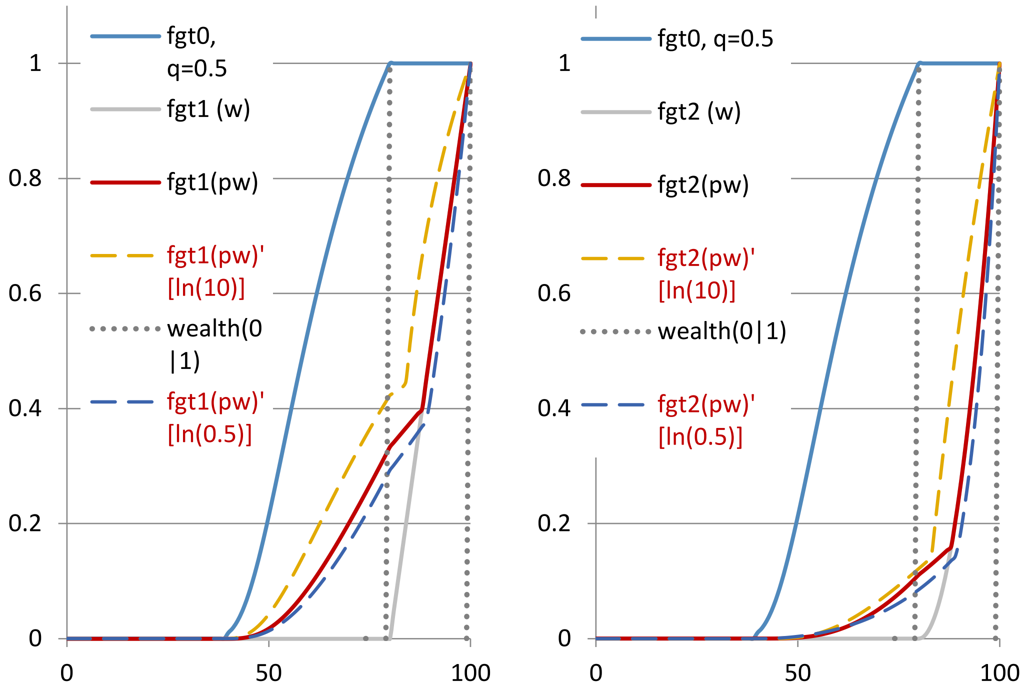
[12] Joint Cardinal FGT Measures for Intensity (gaps) of Poverty and Partial Deprivation*



*here: $i(u)_i = \left(\frac{1}{1+\alpha}\right)^{\alpha} \left(\frac{u - y_i}{u}\right)^{\alpha}$ $for\ y_i < u$

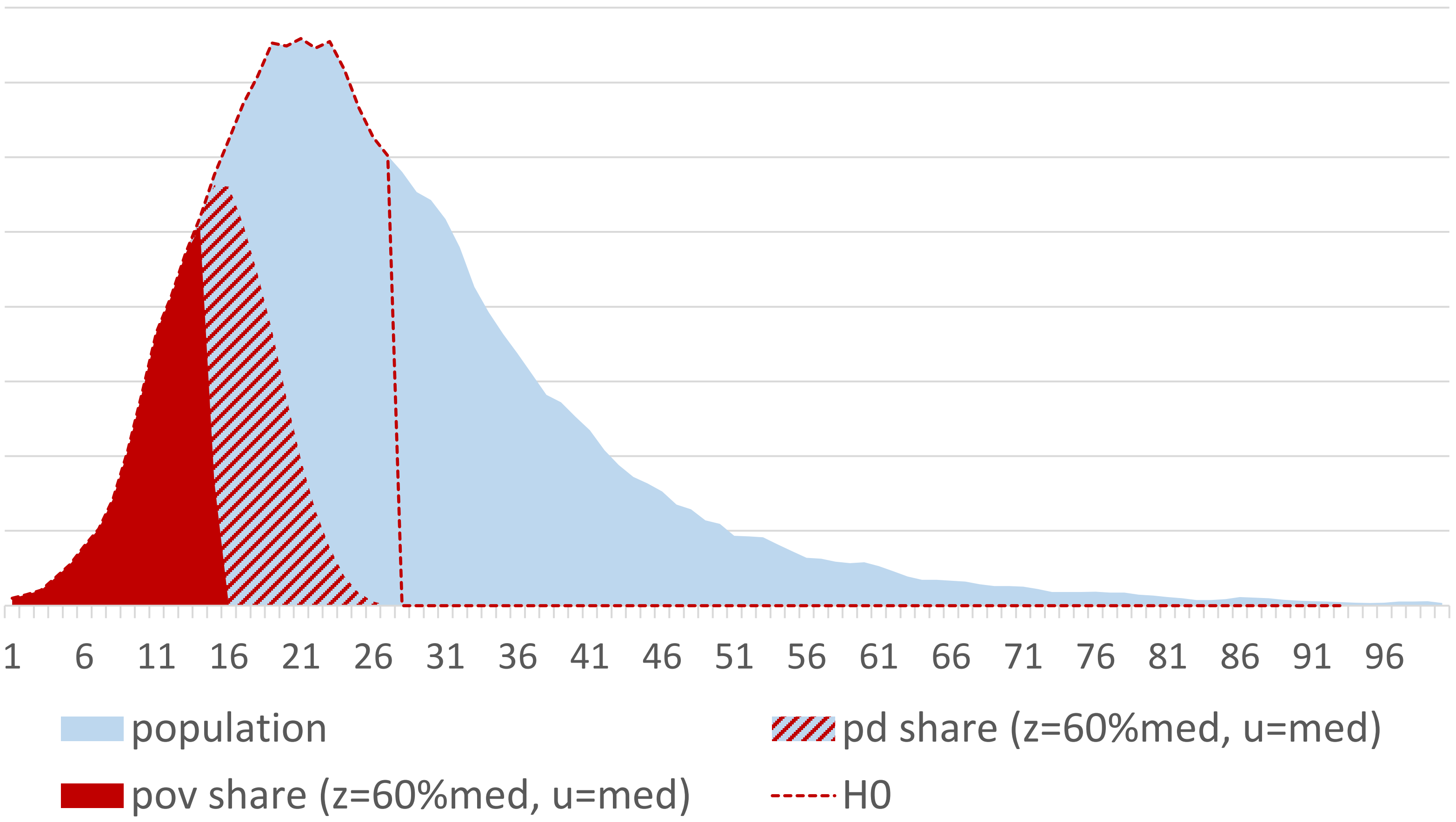
Instead of: $i(u)_i = b^{-a*\alpha} \left(\frac{u - y_i}{u - dmin}\right)^{\alpha}$ $for\ y_i < u$

[12] Joint Cardinal FGT Measures for Intensity (negative gaps) of Wealth and Partial Wealth



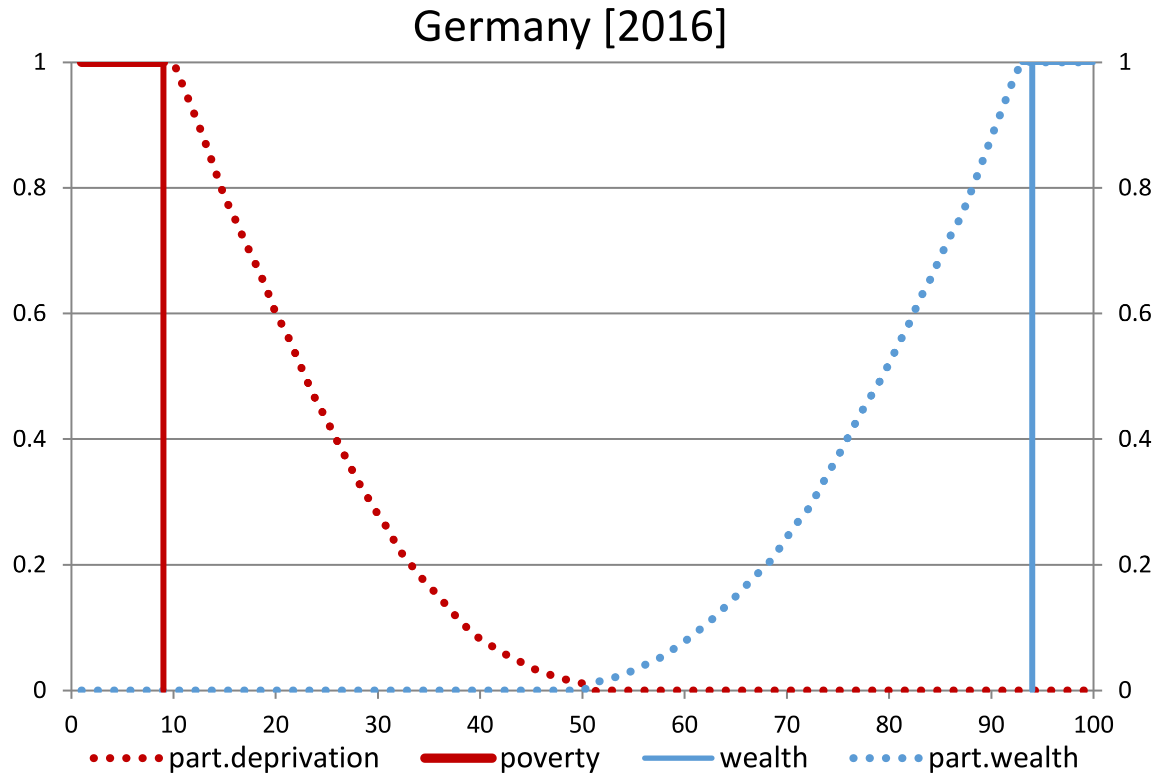
$i(l)_i = q^{a*\alpha} \left(\frac{x_i - l}{d_{max} - l}\right)^{\alpha}$ $for\ x_i > l$

Distribution of conventional income poverty rates (60%med) and related partial deprivation rates in Germany, 2009-2013



SOEPv31l (2010-2013); z=60%med. pd: τ=2.0246, b=5/3, u=med, lim_u=0.01.

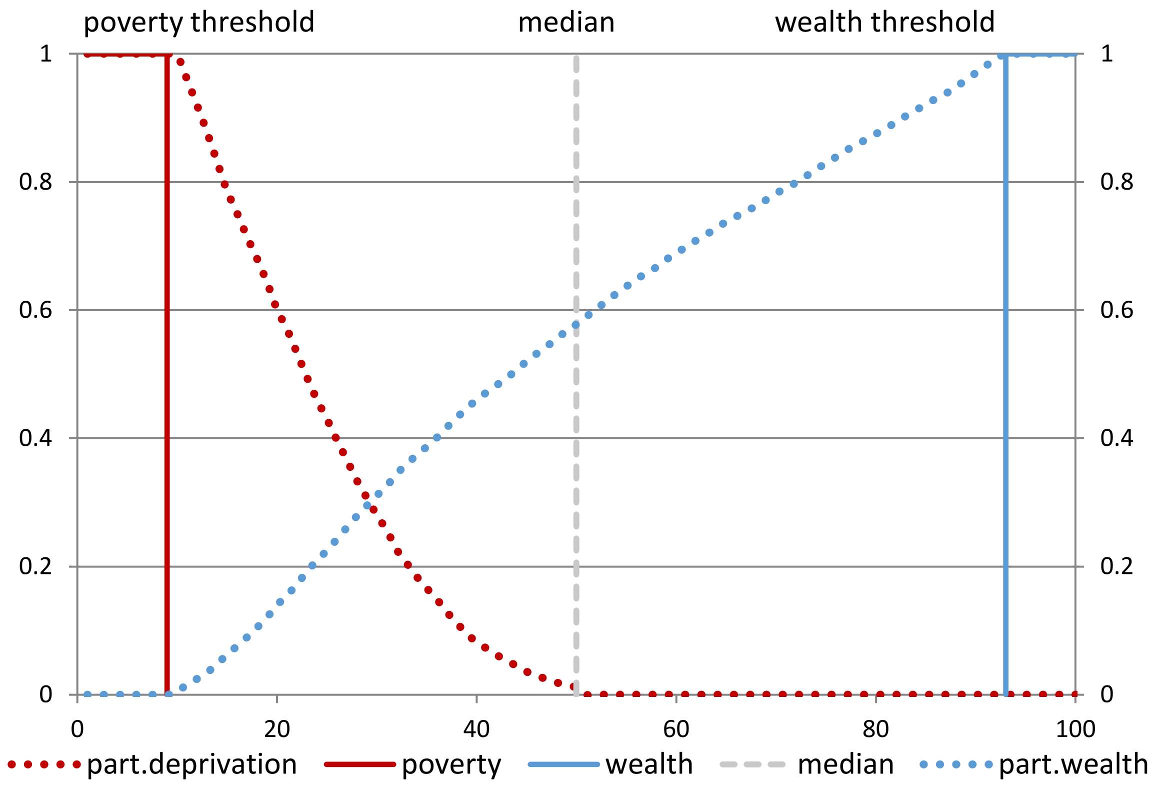
… Rich versus Poor …



Standard Approach –
Concepts for Rich and
Poor are exclusive

… Rich *and* Poor …

Partial Approach –
Concepts for Rich and
Poor may overlap



M | K | GG GINI Indices

[3] Multidimensional | K-dimensional GINI Inequality Indices [M|K|G.. , M|K|GG..]

3.1 Gini Index – calculation | within-between-group notification | population adjustments

3.2 M|K-dimensional Gini SubIndices - (decomposition by k-group | subgroup | dimension)

3.3 Horizontal Inequalities – (Diversities) | inequality notions)

GINI Indices

Mukhopadhyay/Sengupta 2021; Jedrzejczak 2008; Larraz et al. 2021

Gini-Index

Gini, Corrado (1936)

- statistical (empirical) notation
- geometric (Lorenz-type) notation
- covariance notation

Inequality *indicates differences in living conditions as tensions along the (vertical) rich-poor dimension,*

whereas ***Diversity*** *reflects (horizontal) between-group variations in ways of living.*

(1a) $$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2N\sum_{i=1}^{n}x_i}$$ with $\sum_{i=1}^{n} = N$ and $\sum_{j=1}^{n} = N$

(1b) $$G = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}|x_i - x_j|}{2(N-1)\sum_{i=1}^{n}x_i}$$ with $\sum_{i=1}^{n} = N$ and $\sum_{j=1}^{n} = N$

(2) $$GG = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(|x_i - x_j|) * w_i * w_j}{2(N-w_i)\sum_{i=1}^{n}x_{wi}}$$ with $\sum_{i=1}^{n} w_i = N$ and $\sum_{j=1}^{n} w_j = N$

If we recall the block of absolute individual differences in (2) for variable x as

(3) $$Gsum(x_{wi}) = \sum_{i=1}^{n}\sum_{j=1}^{n}(|x_i - x_j|) * w_i * w_j$$

and the total sum of population weighted x in population N as

(4) $$Xsum(x_{wi}) = \sum_{i=1}^{N} x_i * w_i \ ,$$

we can simplify the adjusted standard formulation of the Gini index as

(5) $$GG_i = \frac{Gsum(x_{wi})}{2(N-w_i) * Xsum(x_{wi})/\mathrm{N}} = \frac{Gsum(x_{wi})/(N-w_i)}{2 * Xsum(x_{wi})/\mathrm{N}} \ ,$$

and the aggregated Gini index as mean of individual Gini inequality scores

(6) $$GG = \sum_{i=1}^{n} GG_i \ / \ N \ .$$

Calculation of Gini (Sub-)Indices

							n_j	1	2	3	4	5	6	7	8	9	10				
							l	1	1	1	2	2	2	2	3	3	3				
							x_j	0	10	20	0	10	20	30	10	20	30				
							w_j	0,5	1,0	1,5	0,5	1,0	1,0	1,5	0,5	1,0	1,5				
							wx_j	0	10	30	0	10	20	45	5	20	45				
							wx_l	13	13	13	19	19	19	19	23	23	23				
n_i	k	k_n	x_i	w_i	wx_i	wx_k	GG														
1	1	3	0	0,5	0	13,33		0	5	15	0	5	10	23	2,5	10	23	92,5	20	72,5	25,8
2	1	3	10	1,0	10	13,33		5	0	15	5	0	10	30	0	10	30	105	20	85	51,7
3	1	3	20	1,5	30	13,33		15	15	0	15	15	0	23	7,5	0	23	113	30	82,5	77,5
4	2	4	0	0,5	0	18,75		0	5	15	0	5	10	23	2,5	10	23	92,5	37,5	55	15
5	2	4	10	1,0	10	18,75		5	0	15	5	0	10	30	0	10	30	105	45	60	30
6	2	4	20	1,0	20	18,75		10	10	0	10	10	0	15	5	0	15	75	35	40	30
7	2	4	30	1,5	45	18,75		23	30	23	23	30	15	0	15	15	0	173	67,5	105	45
8	3	3	10	0,5	5	23,33		2,5	0	7,5	2,5	0	5	15	0	5	15	52,5	20	32,5	24,2
9	3	3	20	1,0	20	23,33		10	10	0	10	10	0	15	5	0	15	75	20	55	48,3
10	3	3	30	1,5	45	23,33		23	30	23	23	30	15	0	15	15	0	173	30	143	72,5
							GMK														
10	3		150	10	185	185		0	0	0	1,4	2,7	2,7	4,1	2,5	5	7,5	1055	325	730	420
								0	0	0	2,7	5,4	5,4	8,1	5	10	15				
n	Gini		G		0,285			0	0	0	4,1	8,1	8,1	12	7,5	15	23	GG	GK	GD	GMK
n-1	Gini		G		0,317			1,4	2,7	4,1	0	0	0	0	1,1	2,3	3,4				
								2,7	5,4	8,1	0	0	0	0	2,3	4,6	6,9				
GG	GK	GD	GMK					2,7	5,4	8,1	0	0	0	0	2,3	4,6	6,9				
GG	GGK	GGD	GGMK					4,1	8,1	12	0	0	0	0	3,4	6,9	10				
								2,5	5	7,5	1,1	2,3	2,3	3,4	0	0	0				
0,32	0,10	0,22	0,13					5	10	15	2,3	4,6	4,6	6,9	0	0	0				
0,32	0,36	0,30	0,17					7,5	15	23	3,4	6,9	6,9	10	0	0	0				

Within-group and between-group differentiations of the Gini coefficient

(7) $$GG = GK + GD\ .$$

Gini - within-group index

(8) $$GK = \frac{\sum_{ki=1}^{n}\sum_{lj=1}^{n}(|x_{ki}-x_{lj}|)*w_i*w_j}{2(N-w_i)\sum_{i=1}^{n}x_{wi}}$$ for l = k .

Gini - between-group index

(9) $$GD = \frac{\sum_{ki=1}^{n}\sum_{lj=1}^{n}(|x_{ki}-x_{lj}|)*w_i*w_j}{2(N-w_i)\sum_{i=1}^{n}x_{wi}}$$ for l ≠ k .

we simplify within-group term GK and between-group component GD as

(10) $$GK_i = \frac{GKsum(x_{wki})}{2(N-w_i)*Xsum(x_{wi})} = \frac{GKsum(x_{wki})/(N-w_i)}{2*Xsum(x_{wi})/N}$$ for l = k

(12) $$GD_i = \frac{GDsum(x_{wli})}{2(N-w_i)*Xsum(x_{wi})} = \frac{Gdsum(x_{wli})/(N-w_i)}{2*Xsum(x_{wi})/N}$$ for l ≠ k .

Aggregated within-between-group components can be derived as means

(14) $$GK = \sum_{i=1}^{n} GK_i\ /\ N$$ for l = k and

(15) $$GD = \sum_{i=1}^{n} GD_i\ /\ N$$ for l ≠ k .

Can we decompose Gini …

[factor decomposition]

- by within-between-group inequalities → yes
- by adressing inequalities to subgroups (without overlaps) → no

[subgroup decomposition]

- by subgroup levels → yes

Yitzhaki 1994; Yitzhaki/Lerman 1991; Dagum 1980, Deutsch/Silber 1997; 1997a,b; Georgi 1999; Radaelli 2010; Costa 2008, 2021;

Mean-Group-Differences as between-group Gini sub-index for Horizontal Inequality

Variations in GD describe any absolute differences in variable x between individual i at group level k (k=l) and any other individual j at any other subgroup levels l (k≠l) . The variations in GD between individuals i and j also include mean group differences between the own subgroup level k and any other subgroup level l. To obtain further information on the sources of between-group differences, we treat mean group differences GMK as an integral part of overall GD components.

$$(13) \qquad GMK = \frac{\sum_{ki=1}^{n} \sum_{lj=1}^{n} (|\mu_k(x_{ki}) - \mu_l(x_{lj})|) * w_i * w_j}{2(N-w_i) \sum_{i=1}^{n} x_{wi}} \qquad \text{with } \sum_{i=1}^{n} w_i = N \text{ and } \sum_{j=1}^{n} w_j = N.$$

we accordingly simplify inequalities in mean group differences GMK as

$$(16) \qquad GMK_i = \frac{GMKsum(x_{wli})}{2(N-w_i) * Xsum(x_{wi})/N} = \frac{GMKsum(x_{wli})/(N-w_i)}{2 * Xsum(x_{wi})/N} \quad .$$

Aggregated Gini based Mean group inequalities are only defined for between group operations and are regarded as horizontal inequality measures

$$(17) \qquad GMK = \sum_{i=1}^{n} GMK_i \;/\; N \qquad \text{for } l \neq k .$$

Bhattacharya/Mahalanobis 1967; Camelas/Gisselquist 2018,2019; Stewart 2005, 2009; Stewart et al. 2005, 2010.

Population Adjustments for Within- and Between-Group Gini SubIndices

Gini – raw indices		**Gini population adjusted indices**		**Total**	
GG	$\frac{Gsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	*GG*	$\frac{Gsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$		Total Inequalities
GK	$\frac{GKsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	GGK	$\frac{GKsum(x_{wi})/(Nk-w_i)}{2*Xsum(x_{wi})}$	for l = k	within-group I.
GD	$\frac{GDsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	GGD	$\frac{GDsum(x_{wi})/(N-Nk)}{2*Xsum(x_{wi})}$	for l ≠ k	between-group I.
GMK	$\frac{GMKsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	GGMK	$\frac{GMKsum(x_{wi})/(N-Nk)}{2*Xsum(x_{wi})}$	for l ≠ k	between-group I.
GG = GK + GD					decomposition

M|K-dimensional Gini SubIndices

M|K-dimensional Applications of Gini Indices

Row-first approach by Decanq and Lugo (2012)

	K-Groups [H]	
	one-dimensional	*k-dimensional*
Dimensions [D]		
one-dimensional	***G.. | GG..***	***KG.. | KGG ..***
m-dimensional	***MG.. | MGG ..***	***MKG.. | MKGG ..***

G.. raw Gini indices; *GG..* population adjusted Gini indices.

Dimensional weights sum up to 1

(20) $$\sum_{s=1}^{d} dw_s = 1 \,.$$

(21) $$MGG = \frac{\sum_{s=1}^{d} dw_s \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\left(\left| x_{si} - x_{sj} \right| \right) * w_{si} * w_{sj} \right)}{2(N - w_i) \sum_{s=1}^{d} dw_s \sum_{i=1}^{n} * x_{wsi}}$$

for $\sum_{i=1}^{n} w_i = \sum_{j=1}^{n} w_j = N$ and $\sum_{s=1}^{d} dw_s = 1$

(26) $$KGG = \frac{\sum_{k=1}^{h} kdw_h \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\left(\left| x_{ki} - x_{kj} \right| \right) * w_i * w_j \right)}{2(N - w_i) \sum_{k=1}^{h} kdw_h \sum_{i=1}^{n} * x_{wi}}$$

for $\sum_{i=1}^{n} w_i = \sum_{j=1}^{n} w_j = N$ and $\sum_{k=1}^{h} kdw_h = 1$

(27) $$MKGG = \frac{\sum_{s=1}^{d} kdw_{sh} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\left(\left| x_{ki} - x_{kj} \right| \right) * w_i * w_j \right)}{2(N - w_i) \sum_{s=1}^{d} kdw_{sh} \sum_{i=1}^{n} * x_{wi}}$$

for $\sum_{i=1}^{n} w_i = \sum_{j=1}^{n} w_j = N$

and $\sum_{s=1}^{d} kdw_{sh} = 1$ with $\sum_{s=1}^{d} kdw_{sh} = \sum_{s=1}^{d} \sum_{k=1}^{h} dw_s * kdw_{sh} = 1$ as a special case.

Population Adjustments for M|K-dimensional Gini SubIndices

Gini – raw indices		Gini population adjusted indices		Total
M|K|G	$\frac{\mathrm{M\|K\|}Gsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	M|K|*GG*	$\frac{\mathrm{M\|K\|}Gsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	Total Inequalities
M|K|GK	$\frac{\mathrm{M\|K\|}GKsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	M|K|GGK	$\frac{\mathrm{M\|K\|}GKsum(x_{wi})/(Nk-w_i)}{2*Xsum(x_{wi})}$	within-group I.
M|K|GD	$\frac{\mathrm{M\|K\|}GDsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	M|K|GGD	$\frac{\mathrm{M\|K\|}GDsum(x_{wi})/(N-Nk)}{2*Xsum(x_{wi})}$	between-group I.
M|K|GMK	$\frac{\mathrm{M\|K\|}GMKsum(x_{wi})/(N-w_i)}{2*Xsum(x_{wi})}$	M|K|GGMK	$\frac{\mathrm{M\|K\|}GMKsum(x_{wi})/(N-Nk)}{2*Xsum(x_{wi})}$	between-group I.
M|K|G = M|K|GK + M|K|GD				Decomp.

Horizontal Inequalities

Between-Group-Related Decompositions of the Gini Coefficient I

(16) $$\boldsymbol{GD = GMK + GDD - GDQ}\ .$$

The subindices GDD and GDQ summarize the joint within-group differences to mean group levels as the net effect of absolute differences using $xx_i = \min(x_{ki},x_{lj})$, $xx_k = \min(x_k,x_l)$, $xx_j = \max(x_{ki},x_{lj})$, $xx_l = \max(x_k,x_l)$, with GDD for positive differences and GDQ for overlapping terms as negative differences

(17) $$GDD = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_i - xx_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_j - xx_l|)*w_i*w_j)}{2(N-w_i)\sum_{k=1}^{k}\sum_{i=1}^{n} x_{wki}}$$

for $xx_i \leq xx_k$, $xx_l \leq xx_j$, and $l \neq k$

(18) $$GDQ = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_i - xx_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_j - xx_l|)*w_i*w_j)}{2(N-w_i)\sum_{k=1}^{k}\sum_{i=1}^{n} x_{wki}}$$

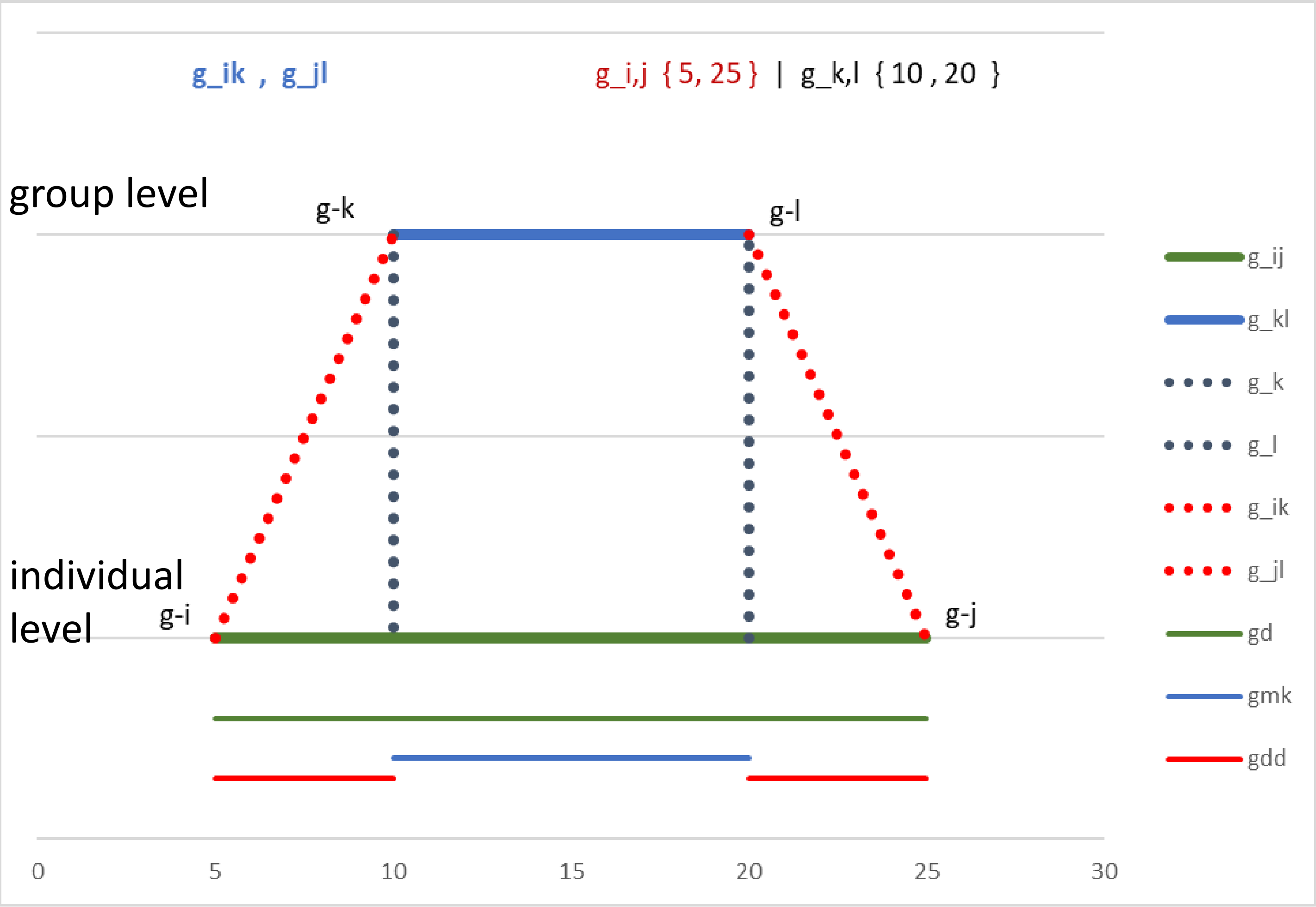
for $xx_i > xx_k$, $xx_l > xx_j$, and $l \neq k$.

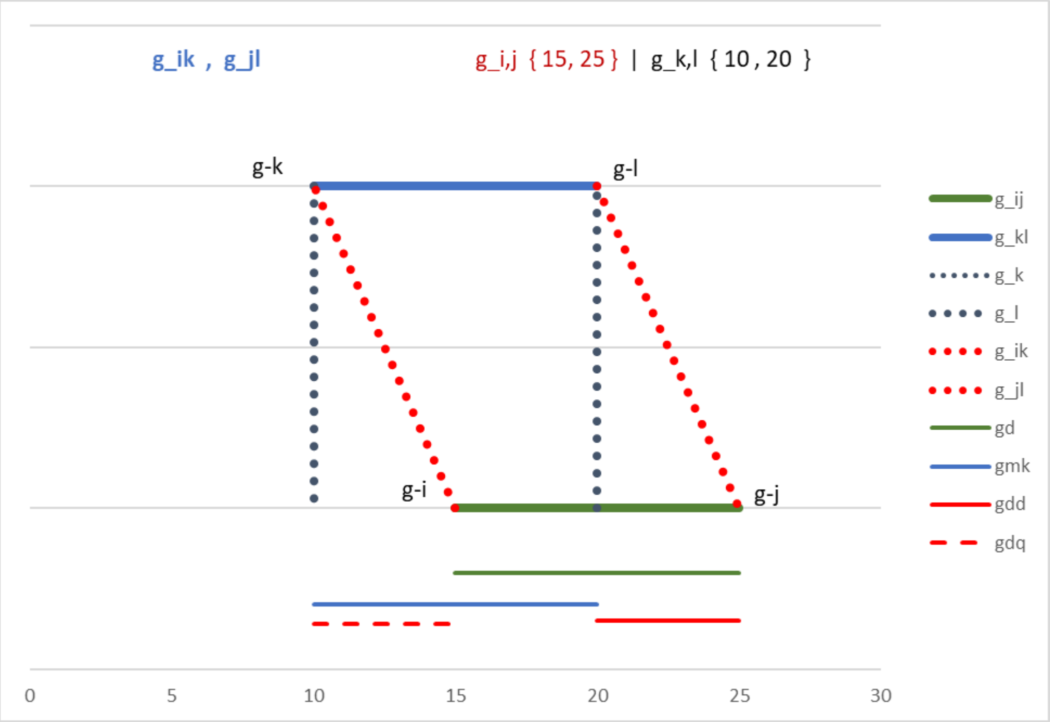
Both indices can again be simplified accordingly:

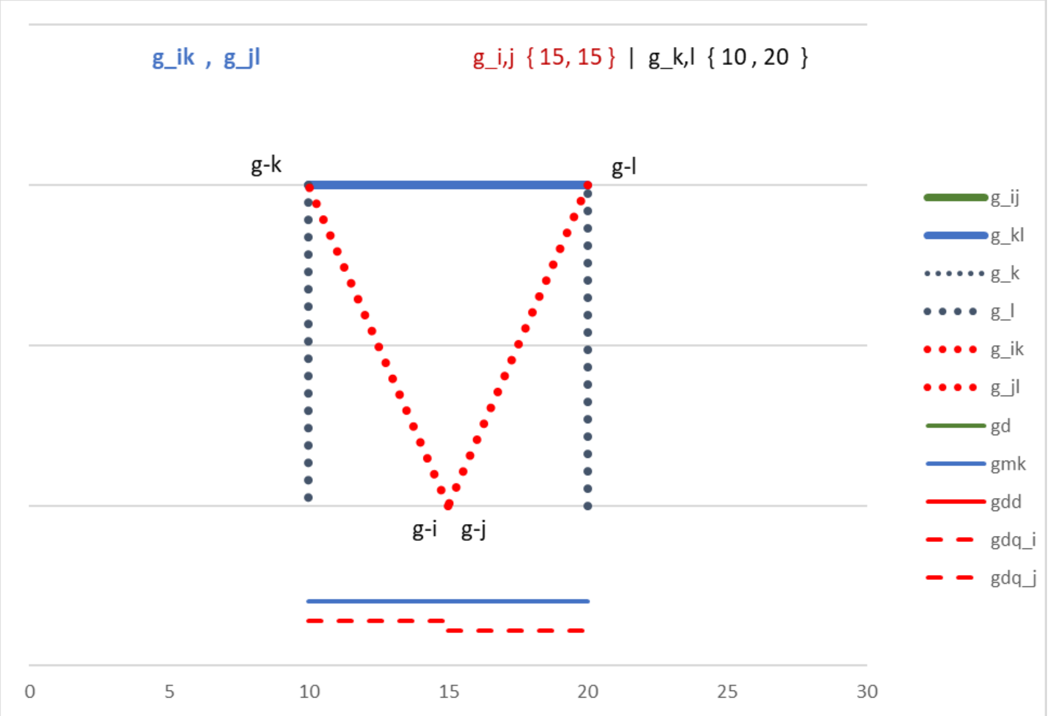
(21) $$GDD = \frac{GDDsum(x_{wli})}{2(N-w_i)*Xsum(x_{wi})} = \frac{GDDsum(x_{wli})/(N-w_i)}{2*Xsum(x_{wi})}$$, for $xx_i \leq xx_k$, $xx_l \leq xx_j$, and $l \neq k$ and

(22) $$GDQ = \frac{GDQsum(x_{wli})}{2(N-w_i)*Xsum(x_{wi})} = \frac{GDQsum(x_{wli})/(N-w_i)}{2*Xsum(x_{wi})}$$, for $xx_i > xx_k$, $xx_l > xx_j$, and $l \neq k$.

Between-Group-Related Decompositions of the Gini Coefficient







$$GD = GMK + GDD - GDQ$$

Between-Group-Related Decompositions of the Gini Coefficient II

(23) $$GD = GMK + GDDR - GDQR - GDQM\ .$$

This version has to consider two further kinds of operations—for $xk_i > x_{lj}$ at the individual level and for $x_k > x_l$ at the group level—which were both excluded by definition in the net operations due to the min|max option in the first version. For $x_i > x_j$ any relations in $x_{ki} \leq x_k \leq x_l \leq x_{lj}$ are reversed as $x_{ki} > x_l$, $x_k > x_l$, $x_l > x_{lj}$ to avoid negative sums.

(24) $$GDDR = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_{ki}-x_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_{lj}-x_l|)*w_i*w_j)}{2(N-w_i)\sum_{k=1}^{k}\sum_{i=1}^{n}x_{wki}}$$

for $[x_i \leq x_j$ and $x_{ki} \leq x_k$, $x_l \leq x_{lj}]$ or $[x_i > x_j$ and $x_{ki} > x_k$, $x_l > x_{lj}]$, and $l \neq k$

(25) $$GDQR = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_{ki}-x_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_{lj}-x_l|)*w_i*w_j)}{2(N-w_i)\sum_{k=1}^{k}\sum_{i=1}^{n}x_{ki}}$$

for $[x_i \leq x_j$ and $x_{ki} > x_k$, $x_l > x_{lj}]$ or $[x_i > x_j$ and $x_{ki} \leq x_k$, $x_l \leq x_{lj}]$, and $l \neq k$.

For $x_k > x_l$ we add—just as we do for GDD(R) and GDQ(R) on the individual level—an overlapping term at group level GDQM as a counterpart to the absolute mean group differences GMK.

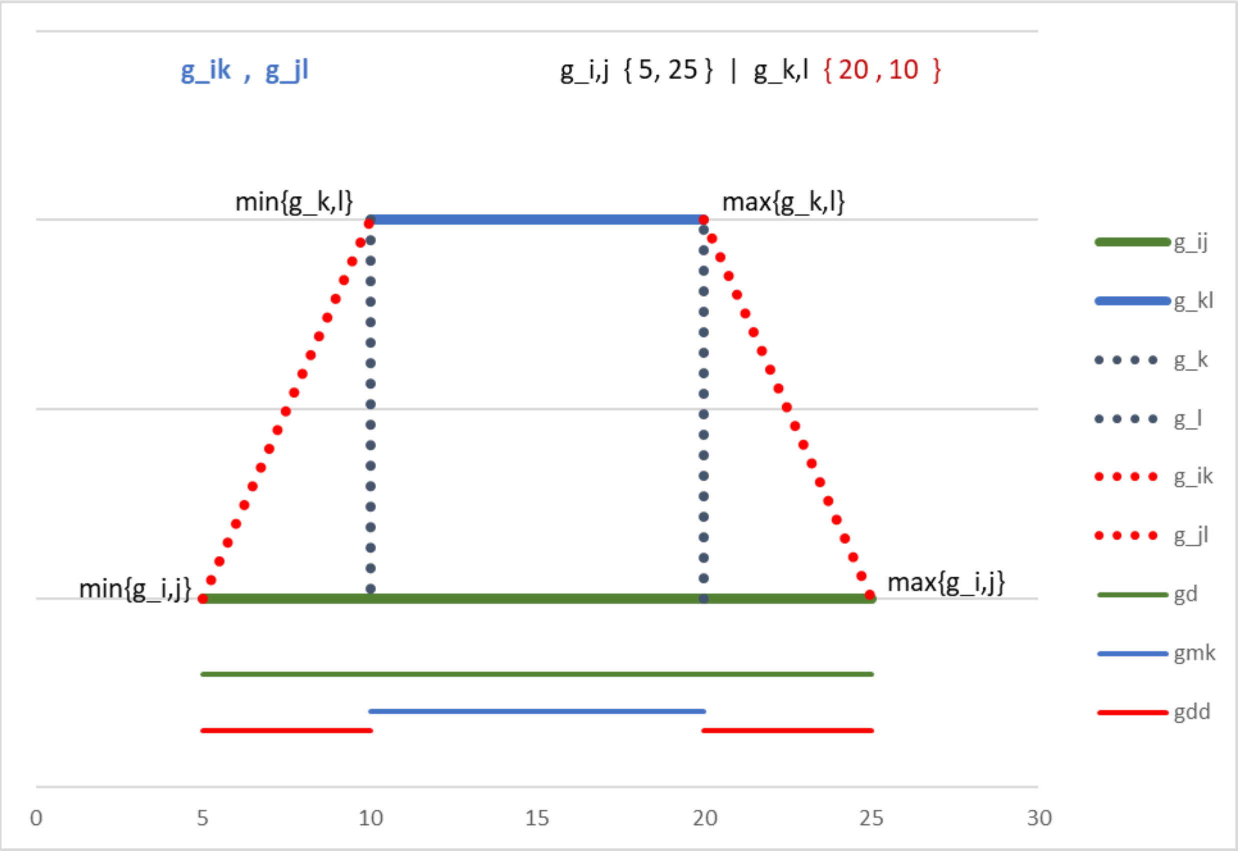
(30) $$GDQM = 2 * \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(|\mu_k(x_{ki}) - \mu_l(x_{lj})|)*w_i*w_j}{2(N-w_i)\sum_{i=1}^{n}x_{wi}}$$

for $[x_i \leq x_j$ and $x_k < x_l]$ or $[x_i > x_j$ and $x_k \geq x_l]$, and $l \neq k$.

Between-Group-Related Decompositions of the Gini Coefficient

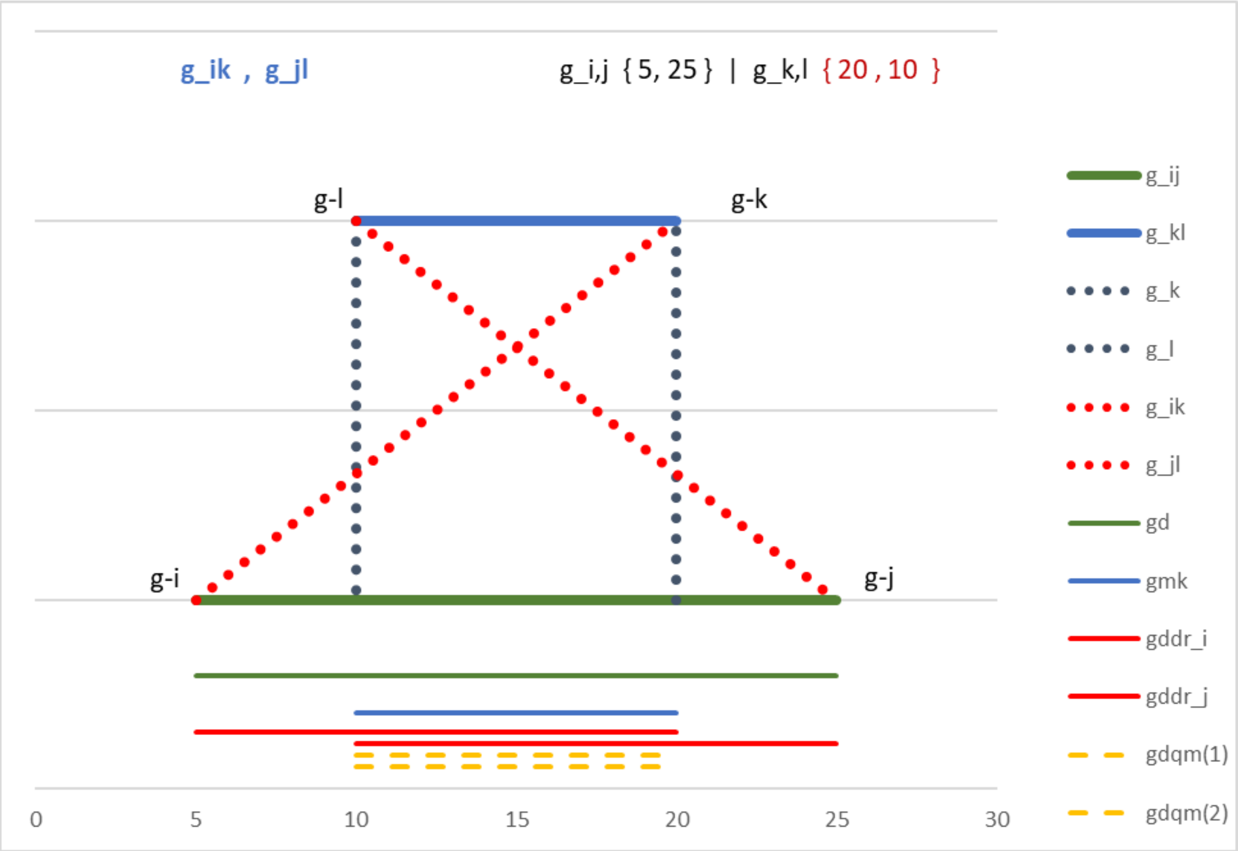
Version I

$$GD = GMK + GDD - GDQ$$



Version II

$$GD = GMK + GDDR - GDQR - GDQM$$



Diversity Indices - Between-Group-Related Gini sub-indices

For decomposition of between-group inequalities according to mean group levels, we mentioned two opposing joint individual within-group differences: GDD vs. GDQ for Version 1 and GDDR vs. GDQR for Version 2. Each of these components, which are all non-integral parts of the original Gini calculation, are sensitive to joint within-group variations, referring to each element of absolute individual differences across subgroups. GDD and GDQ (as well as GDDR and GDQR) together describe the full amount of joint within-group variation net of differences in mean group levels for each element of Gini calculation in different subgroups.

We indicate these inequality-based joint variations within different subgroup levels as diversity measures. For the calculation of Gini-based diversity indices, we use both versions of joint within-group differences to mean group levels (GDD, GDQ and GDDR, GDQR).

The resulting Gini-based diversity indices **GDIV1** and **GDIV2** can thus be regarded as further Gini-based subindices as well.

(33) $$GDIV1 = GDD + GDQ \qquad \text{for } l \neq k$$

(37) $$GDIV2 = GDDR + GDQR \qquad \text{for } l \neq k$$

Diversity Indices - GDIV1 GDIV2

GDIV1 may be regarded as a lower bound in overall diversities, assuming full reports of within-group variation by each individual i and j from different subgroups k and l. GDIV1 is conceptualized as the net effect of absolute differences using $xx_i = \min(x_{ki},x_{lj})$, $xx_k = \min(x_k,x_l)$, $xx_j = \max(x_{ki},x_{lj})$, $xx_l = \max(x_k,x_l)$. Differences in individual levels x_{ki} and x_{lj} as well as the related differences in group levels x_k and x_l are fully recognized by each other and therefore interchangeable between any individuals i and j.

(34) $$GDIV1 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_i - xx_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|xx_j - xx_l|)*w_i*w_j)}{2(N-1)\sum_{k=1}^{k}\sum_{i=1}^{n} x_{wki}}$$ for $l \neq k$

(36) $$GDIV1 = \frac{GDDsum(xw_{li}) + GDQsum(x_{wli})}{2(N-w_i)*Xsum(x_{wi})} = \frac{GDIV1sum(x_{wli})/(N-w_i)}{2*Xsum(x_{wi})}$$ for $l \neq k$

GDIV2 marks upper bounds in inequality-based diversities between each pair of individuals i and j from different subgroups k and l. This specification assumes that each of the individuals refers primarily to their own subgroup level and takes further notice of variations in subgroup-related differences of other individuals to their subgroup level.

(38) $$GDIV2 = \frac{\sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_{ki} - x_k|)*w_i*w_j) + \sum_{i=1}^{n}\sum_{j=1}^{dn}((|x_j - x_{lj}|)*w_i*w_j)}{2(N-w_i)\sum_{k=1}^{k}\sum_{i=1}^{n} x_{wi}}$$ for $l \neq k$

(40) $$GDIV2 = \frac{GDDRsum(x_{wli}) + GDQRsum(x_{wli})}{2(N-w_i)*Xsum(x_{wi})} = \frac{GDIV2sum(x_{wli})/(N-w_i)}{2*Xsum(x_{wi})}$$ for $l \neq k$

Population Adjustments for Gini based Diversity Indices

Gini – raw Diversity indices		**Gini pop. adj. Diversity indices**		**Between Group**	
GDIV1	$\frac{GDIV1sum(x_{wi})/(N-1)}{2 * Xsum(x_{wi})}$	*GGDIV1*	$\frac{GDIV1sum(x_{wi})/(N-Nk)}{2 * Xsum(x_{wi})}$	for l ≠ k	Diversity Index I [net effect]
GDIV2	$\frac{GDIV2sum(x_{wi})/(N-1)}{2 * Xsum(x_{wi})}$	*GGDIV2*	$\frac{GDIV2sumx_{wi})/(N-Nk)}{2 * Xsum(x_{wi})}$	for l ≠ k	Diversity Index II [total effect]
GDIV1 = GDD + GDQ		*GGDIV1 = GGDD + GGDQ*		for l ≠ k	Decomposition I
GDIV2 = GDDR + GDQR		*GGDIV2 = GGDDR + GGDQR*		for l ≠ k	Decomposition II

M|K-dimensional Gini-based (Sub-)Indices

For multidimensional applications of Gini subindices and diversity indices, we follow the **row-first approach** by Decanq and Lugo (2012) and others. Any Gini index can, according to this approach, be reassigned as a multidimensional index using dimensional weighting with sum of dimensional weights (dw) = 1. The dimensional weights specify the percentage to which each dimension (or variable) s contributes to the supposed overall inequality across dimensions.

Maasoumi 1986; Decanq/Lugo 2012

A standard approach is to use equal weights or any design-based variable assignments. We also propose the use of welfare-based individual weighting schemes, as dimensional priorities may vary between individuals and may change during the life cycle.

Any of the previously differentiated raw [G.. {GK,GD,GKM,GDD*,GDQ*}] and population-adjusted [GG.. {GGK,GGD,GGKM,GGDD*,GGDQ*}] Gini subindices can accordingly be assigned for multidimensional applications as **M|K|G…** and **M|K|GG…** .

Programs

[4] Programs and tools for multidimensional well-being applications

4.1 MPPIS | MWWIS – Multidimensional Deprivation and WellBeing

4.2 MGDIV – Multidimensional Inequalities [Gini]

4.3 Tools – Dimensional Weighting | Rescaling

Programs MPPIS | MWWIS – multi-dimensional Deprivation & WellBeing

```
program define mppis, eclass
syntax varlist          [if] [in] [pweight aweight iweight], z(varlist)    [ zeq(numlist)          ///
                         af  af_k(numlist)  ri ri_th(numlist max=1) wdim(varlist)   partial          ///
                         bs bs_strata(varlist max=1) bs_rep(numlist max=1) bs_seed(numlist max=1) svy  ///
                         DECOMPosition SUBGroup(varlist max=1) GROUPdecomp(varlist max=1)          ///
                         eps(varlist) tau(varlist) b(varlist) u(varlist) min(varlist) lim(varlist)   ///
                         lim2(varlist) u2(varlist) eps2(varlist) /*2. Segment*/                     ///
                         lim3(varlist) u3(varlist) eps3(varlist) /*3. Segment*/                     ///
                         m_a(numlist max=1) mpd_b(numlist) mpd_x(numlist)                           ///
                         md_log md_logbase(numlist) mpd_log mpd_logbase(numlist)                    ///
                         details  noresult inputm0  cardinal                                       ///
```

```
program define mwwis, eclass
syntax varlist          [if] [in] [pweight aweight iweight], w(varlist)    [ weq(numlist)          ///
                         af  af_k(numlist)  ri ri_th(numlist max=1) wdim(varlist)   partial          ///
                         bs bs_strata(varlist max=1) bs_rep(numlist max=1) bs_seed(numlist max=1) svy  ///
                         DECOMPosition SUBGroup(varlist max=1) GROUPdecomp(varlist max=1)          ///
                         eps(varlist) tau(varlist) q(varlist) l(varlist) max(varlist) lim(varlist)   ///
                         lim2(varlist) l2(varlist) eps2(varlist) /*2. Segment*/                     ///
                         lim3(varlist) l3(varlist) eps3(varlist) /*3. Segment*/                     ///
                         m_a(numlist max=1) mpw_q(numlist) mpw_x(numlist)                           ///
                         mw_log mw_logbase(numlist) mpw_log mpw_logbase(numlist)                    ///
                         details  noresult inputm0  cardinal                                       ///
```

MPPIS - multi-dimensional Deprivation

```
program define mppis, eclass
syntax varlist      [if] [in] [pweight aweight iweight], z(varlist)    [ zeq(numlist)           ///
                    af  af_k(numlist)   ri ri_th(numlist max=1) wdim(varlist)    partial          ///
                    bs bs_strata(varlist max=1) bs_rep(numlist max=1) bs_seed(numlist max=1) svy   ///
                    DECOMPosition SUBGroup(varlist max=1) GROUPdecomp(varlist max=1)               ///
                    eps(varlist) tau(varlist) b(varlist) u(varlist) min(varlist) lim(varlist)     ///
                    lim2(varlist) u2(varlist) eps2(varlist) /*2. Segment*/                         ///
                    lim3(varlist) u3(varlist) eps3(varlist) /*3. Segment*/                         ///
                    m_a(numlist max=1) mpd_b(numlist) mpd_x(numlist)                               ///
                    md_log md_logbase(numlist) mpd_log mpd_logbase(numlist)                        ///
                    details  noresult inputm0  cardinal                                            ///

                    vmd_h   vmd_h1  vmd_h0  vmd_md   vmd_hc  vmd_m0c  vmd_m1c  vmd_mac              ///
                    vmpd_h vmpd_h1 vmpd_h0 vmpd_md  vmpd_hc vmpd_m0c vmpd_m1c vmpd_mac              ///
                    vmd_m0th vmd_m1th vmd_math vmpd_m0th vmpd_m1th vmpd_math                        ///
                    vmd_h_name(string)  vmd_h1_name(string)  vmd_h0_name(string)  vmd_md_name(string)         ///
                    vmpd_h_name(string) vmpd_h1_name(string) vmpd_h0_name(string) vmpd_md_name(string)        ///
                    vmd_hc_name(string)  vmd_m0c_name(string)  vmd_m1c_name(string)  vmd_mac_name(string)     ///
                    vmpd_hc_name(string) vmpd_m0c_name(string) vmpd_m1c_name(string) vmpd_mac_name(string)    ///
                    vmd_m0th_name(string) vmd_m1th_name(string) vmd_math_name(string)                         ///
                    vmpd_m0th_name(string) vmpd_m1th_name(string) vmpd_math_name(string)                      ///
                    vdim_d0(string) vdim_d1(string) vdim_da(string)                                 ]
```

Multidimensional Deprivation MPI[pd] z50 u50 cc=.333 -- syear = 2021

Results for Alkire-Foster dual cutoff procedure

Number of dimensions d:	5
Dimensions:	yinc yinc_ir myinczp minc minc_ehc
Respective cutoffs c_j:	677.9119
Poverty cutoffs k:	33.3
Sampling weights:	pweight = phrf1
Number of observations:	26566

results1[3,4]: General Results for mdH0 mdH1 MD

	[d]H(union)	[d]H1(ints~)	[d]H0(ints~)	[d]MD(Mean~)
Value	.18119175	.03326327	.81880825	.09911304
lower_Value	.17278964	.02961435	.81040613	.09398128
upper_Value	.18959387	.03691219	.82721036	.1042448

results2[9,1]: Ordinal Results for Alkire | Foster (Dual cutoff) [for

	k=33_3
[d]M0c	.09005412
[d]M0c_cib	.08493271
[d]M0c_cit	.09517553
[d]Hc	.13589716
[d]Hc_cib	.12848184
[d]Hc_cit	.14331247
[d]Ac	.66266376
[d]Ac_cib	.64922556
[d]Ac_cit	.67610195

Partial Deprivation: Parameter Values [for _n==1] (N : option details)]

Deprivation threshold u:	25959.76 27071.67 25959.76 1923.077 1355.824
b = u/z:	2 2 2 2 2
Epsilon:	1 1 1 1 1
Tau:	1.243379 1.243379 1.243379 1.243379 1.243379
Limit Deprivation at threshold u:	.01 .01 .01 .01 .01
Weight of partial deprivation FGT-gaps	1 1 1 1 1

Partial Deprivation General Results for mdH mdH1 mdH0 MD:

pd_results1[3,4]: upper and lower = boundaries of 95% confidence intervall

	[pd]H(union)	[pd]H1(int~)	[pd]H0(int~)	[pd]M(Mean~)
Value	.54432668	.03447071	.45567332	.20340187
Value(ci-)	.53332283	.03077391	.44466946	.19659626
Value(ci+)	.55533054	.03816751	.46667717	.21020749

Partial Deprivation - Ordinal Results for Alkire | Foster (Dual cutoff)

pd_results2[9,1]: upper and lower = boundaries of 95% confidence intervall

	k=33_3
[pd]M0c	.17369222
[pd]M0c_cib	.16669776
[pd]M0c_cit	.18068668
[pd]Hc	.24280709
[pd]Hc_cib	.23344457
[pd]Hc_cit	.25216962
[pd]Ac	.71535068
[pd]Ac_cib	.70550849
[pd]Ac_cit	.72519287

```
 Multidimensional Deprivation MPI[pd] z50 u50 cc=.333 -- syear = 2021

Results for Alkire-Foster dual cutoff procedure

Number of dimensions d:               5
Dimensions:                           yinc yinc_ir myinczp minc minc_ehc
Respective cutoffs c_j:               677.9119
Poverty cutoffs k:                    33.3
Sampling weights:                     pweight = phrfl
Number of observations:               26566

results1[3,4]:  General Results for mdH0 mdH1 MD
                [d]H(union)  [d]H1(ints~)  [d]H0(ints~)  [d]MD(Mean~)
       Value     .18119175     .03326327     .81880825     .09911304
 lower_Value     .17278964     .02961435     .81040613     .09398128
 upper_Value     .18959387     .03691219     .82721036      .1042448

results2[9,1]:  Ordinal Results for Alkire | Foster (Dual cutoff) [for pCutoff=...%]
                k=33_3
     [d]M0c  .09005412
 [d]M0c_cib  .08493271
 [d]M0c_cit  .09517553
      [d]Hc  .13589716
  [d]Hc_cib  .12848184
  [d]Hc_cit  .14331247
      [d]Ac  .66266376
  [d]Ac_cib  .64922556
  [d]Ac_cit  .67610195
```

```
Partial Deprivation: Parameter Values [for _n==1] ( N : option details )]
Deprivation threshold u:                25959.76 27071.67 25959.76 1923.077 1355.824
b = u/z:                                2 2 2 2 2
Epsilon:                                1 1 1 1 1
Tau:                                    1.243379 1.243379 1.243379 1.243379 1.243379
Limit Deprivation at threshold u:       .01 .01 .01 .01 .01
Weight of partial deprivation FGT-gaps 1 1 1 1 1

Partial Deprivation General Results for mdH mdH1 mdH0 MD:

pd_results1[3,4]:  upper and lower = boundaries of 95% confidence intervall
                [pd]H(union)  [pd]H1(int~)  [pd]H0(int~)  [pd]M(Mean~)
        Value     .54432668     .03447071     .45567332     .20340187
Value(ci-)        .53332283     .03077391     .44466946     .19659626
Value(ci+)        .55533054     .03816751     .46667717     .21020749

Partial Deprivation - Ordinal Results for Alkire | Foster (Dual cutoff)

pd_results2[9,1]:  upper and lower = boundaries of 95% confidence intervall
                    k=33_3
    [pd]M0c  .17369222
[pd]M0c_cib  .16669776
[pd]M0c_cit  .18068668
     [pd]Hc  .24280709
 [pd]Hc_cib  .23344457
 [pd]Hc_cit  .25216962
     [pd]Ac  .71535068
 [pd]Ac_cib  .70550849
 [pd]Ac_cit  .72519287
```

details[5,5]: Parameters and deprivation in each dimension

	yinc	yinc_ir	myinczp	minc	minc_ehc
dimwgt(min)	.2	.2	.2	.2	.2
dimwgt(max)	.2	.2	.2	.2	.2
z(min)	12979.88	13535.83	12979.88	961.5385	677.9119
z(max)	12979.88	13535.83	12979.88	961.5385	677.9119
D0	.08799536	.08634987	.12824373	.07009272	.12288351

pd_details[15,5]: Partial Deprivation- Parameters and deprivation in each dimension

	yinc	yinc_ir	myinczp	minc	minc_ehc
dimwgt(min)	.2	.2	.2	.2	.2
dimwgt(max)	.2	.2	.2	.2	.2
z(min)	12979.88	13535.83	12979.88	961.5385	677.9119
z(max)	12979.88	13535.83	12979.88	961.5385	677.9119
u(min)	25959.76	27071.67	25959.76	1923.077	1355.824
u(max)	25959.76	27071.67	25959.76	1923.077	1355.824
b(min)	2	2	2	2	2
b(max)	2	2	2	2	2
tau(min)	1.243379	1.243379	1.243379	1.243379	1.243379
tau(max)	1.243379	1.243379	1.243379	1.243379	1.243379
eps(min)	1	1	1	1	1
eps(max)	1	1	1	1	1
lim(min)	.01	.01	.01	.01	.01
lim(max)	.01	.01	.01	.01	.01
PD0	.20502291	.20031707	.1949229	.19261357	.22413291

Programs MWWIS - **multi-dimensional WellBeing**

```
program define mwwis, eclass
syntax varlist      [if] [in] [pweight aweight iweight], w(varlist)    [ weq(numlist)            ///
                    af  af_k(numlist)  ri ri_th(numlist max=1) wdim(varlist)    partial          ///
                    bs bs_strata(varlist max=1) bs_rep(numlist max=1) bs_seed(numlist max=1) svy    ///
                    DECOMPosition SUBGroup(varlist max=1) GROUPdecomp(varlist max=1)            ///
                    eps(varlist) tau(varlist) q(varlist) l(varlist) max(varlist) lim(varlist)      ///
                    lim2(varlist) l2(varlist) eps2(varlist) /*2. Segment*/                       ///
                    lim3(varlist) l3(varlist) eps3(varlist) /*3. Segment*/                       ///
                    m_a(numlist max=1) mpw_q(numlist) mpw_x(numlist)                             ///
                    mw_log mw_logbase(numlist) mpw_log mpw_logbase(numlist)                      ///
                    details  noresult inputm0  cardinal                                          ///

                    vmw_h   vmw_h1  vmw_h0  vmw_mw   vmw_hc  vmw_m0c  vmw_m1c  vmw_mac            ///
                    vmpw_h vmpw_h1 vmpw_h0 vmpw_mw  vmpw_hc vmpw_m0c vmpw_m1c vmpw_mac            ///
                    vmw_m0th vmw_m1th vmw_math vmpw_m0th vmpw_m1th vmpw_math                     ///
                    vmw_h_name(string)  vmw_h1_name(string)  vmw_h0_name(string)  vmw_mw_name(string)        ///
                    vmpw_h_name(string) vmpw_h1_name(string) vmpw_h0_name(string) vmpw_mw_name(string)      ///
                    vmw_hc_name(string) vmw_m0c_name(string)  vmw_m1c_name(string)  vmw_mac_name(string)    ///
                    vmpw_hc_name(string) vmpw_m0c_name(string) vmpw_m1c_name(string) vmpw_mac_name(string)  ///
                    vmw_m0th_name(string) vmw_m1th_name(string) vmw_math_name(string)                  ///
                    vmpw_m0th_name(string) vmpw_m1th_name(string) vmpw_math_name(string)               ///
                    vdim_w0(string) vdim_w1(string) vdim_wa(string)                              ]
```

MWWIS - multi-dimensional WellBeing

```
program define mwwis, eclass
syntax varlist      [if] [in] [pweight aweight iweight], w(varlist)    [ weq(numlist)         ///
                    af  af_k(numlist)  ri ri_th(numlist max=1) wdim(varlist)   partial        ///
                    bs bs_strata(varlist max=1) bs_rep(numlist max=1) bs_seed(numlist max=1) svy  ///
                    DECOMPosition SUBGroup(varlist max=1) GROUPdecomp(varlist max=1)          ///
                    eps(varlist) tau(varlist) q(varlist) l(varlist) max(varlist) lim(varlist) ///
                    lim2(varlist) l2(varlist) eps2(varlist) /*2. Segment*/                    ///
                    lim3(varlist) l3(varlist) eps3(varlist) /*3. Segment*/                    ///
                    m_a(numlist max=1) mpw_q(numlist) mpw_x(numlist)                          ///
                    mw_log mw_logbase(numlist) mpw_log mpw_logbase(numlist)                   ///
                    details  noresult inputm0  cardinal                                       ///

                    vmw_h   vmw_h1  vmw_h0  vmw_mw   vmw_hc  vmw_m0c  vmw_m1c  vmw_mac        ///
                    vmpw_h vmpw_h1 vmpw_h0 vmpw_mw  vmpw_hc vmpw_m0c vmpw_m1c vmpw_mac        ///
                    vmw_m0th vmw_m1th vmw_math vmpw_m0th vmpw_m1th vmpw_math                  ///
                    vmw_h_name(string)  vmw_h1_name(string)  vmw_h0_name(string)  vmw_mw_name(string)        ///
                    vmpw_h_name(string) vmpw_h1_name(string) vmpw_h0_name(string) vmpw_mw_name(string)       ///
                    vmw_hc_name(string) vmw_m0c_name(string)  vmw_m1c_name(string)  vmw_mac_name(string)     ///
                    vmpw_hc_name(string) vmpw_m0c_name(string) vmpw_m1c_name(string) vmpw_mac_name(string)   ///
                    vmw_m0th_name(string) vmw_m1th_name(string) vmw_math_name(string)                        ///
                    vmpw_m0th_name(string) vmpw_m1th_name(string) vmpw_math_name(string)                     ///
                    vdim_w0(string) vdim_w1(string) vdim_wa(string)                          ]
```

Programs MPPIS [MWWIS with similar options]

```
** multidimensional applications [default: af ]
*  af       [def]                : Alkire, S. and J. Foster (2011). Counting and multidimensional poverty measurement.
*                                  Journal of Public Economics 95(7-8), 476-487.
*  af_k     [def 33.3 {0.333]    : dual-cutoff threshold [pcutoff] - percentage of dimensions to indicate as m-poor

*  ri       []                   : ri, Nicole Rippin (2013). Considerations of Efficiency and Distributive Justice
*                                  in Multidimensional Poverty Measurement. Dissertation(Göttingen).
*  ri_th [def 2.0]               : Theta parameter to shape identification across dimensions

** definitions of lines {dimensional cutoffs} -- mppis vs mwwis
*  z       deprivation line(s)                ||                  w          wealth line(s)
*  u       upper deprivation line(s)          ||                  l          lower wealth line(s)
*  b       ratio u/z {b>1}                    ||                  q          ratio l/w {q<1}

* partial [def (no)]             :   parameter to initialize pd-function
* tau [default gen]              :   parameter for baseline pd-function
* eps [default 1.0]              :   parameter to shape pd-function
* b   [default gen]              :   parameter for ratio u/z in pd-function
* u   [input expected]           :   upper bound for zones of deprivation in pd-function
* lim [default 0.01]             :   supposed pd-value at u {~marginal > 0}
* m_pdb [default 2.0]            :   integration parameter - for standard and partial fgt-gaps
• m_pdx [default 0]              :   integration parameter - to consider only partial fgt-gaps [1: standard gaps = 0]

* eps2 eps3                      :   eps parameter for 2.|3. segment
* u2   u3                        :   u parameter for 2.|3. segment
• lim2 lim3                      :   lim parameter for 2.|3. segment

* min [default 0]                :   min coding - used also for m-fgt-adjustments
* cardinal [def no]              :   option for m-fgt-applications {mdM1,..,mdMa}
* m_a [default 2.0]              :   parameter alpha - for m-fgt-functions -- by default: {mdMa[2]}

* zeq [def 0: var < z]           :   option for z-line { 1: var <= z }
* details [def no]               :   option for listing of parameter settings in pd-functions
```

Programs

MGDIV - multi-dimensional Gini

```
                    syntax varlist     [if] [in] [fweight aweight pweight], [ggg gdd gdiv ddiv gmax gmax1          ///
                                       SUBGroup(varlist max=1)  hgroup(varlist) kdim mkdim nogg                   ///
                                       klevel(string) kklevel(string) wdimh(varlist) wdimhd(varlist) skdecomp kkdecomp ///
                                       mdim wdimd(varlist) mgdecomp kgdecomp mkgdecomp decomp_pop decomp_tot      ///
                                       decomp_pct mbeta beta(numlist max=1) vdelta delta(numlist max=1) vout      ///
                                       vmout vmoutdim vkout vkoutdim vmkout vmkoutdim vpre(string) vsuf(string) ]
// options
// ggg              calculate and display GG [GN] GK GD GMK  GGK GGD GGMK
// gdd              calculate and display decompostions of GD|GDD [GDIV1: GDD GDQ | GDIV2: GDDR GDDQ GDQM | ...]
// gdiv             calculate and display GDIV1,2,... | GGDIV1,2,...
// *ddiv            calculate and display decompostions of GDIV1,2,... | GGDIV1,2,... //not yet implemented
// subgroup(var)    subgroup variable for subgroup decomposition
// hgroup(var)      hgroup variable for factor decomposition (within-between & [mean] group {GK GD GMK | GGK GGD GGMK | GDIV.. } )
// skdecomp         subgroup decomposition by [subgroup]
// kkdecomp         subgroup decomposition(s) by [hgroup(s)]
// mdim             apply multidimensional calculation
// kdim             apply k-dimensional calculation
// mkdim            apply multidimensional and k-dimensional calculation
// wdimd(varlist)   list of variables for m-dimensional weighting [n of elements = ndim]
// wdimh(varlist)   list of variables for k-dimensional weighting [n of elements = nhdim (nk1,nk2,...,nkn)]
// mgdecomp         multidimensional factor decomposition by dimension [varlist]
// kgdecomp         k-dimensional factor decomposition by k-dimension [klist]
// mkgdecomp        multidimensional and k-dimensional factor decomposition by k-dimension and dimension [k-list * varlist]
// decomp_pop       display results of subgroup decomposition by subgroup levels
// decomp_tot       display results of decomposition as absolute contributions [default]
// decomp_pct       display results of decomposition as percentage contributions

// output-options
// vout             save individual m-k-versions [before final aggregation{mean}] of all calculated   single Sub-Indices
// vmout            save individual   k-versions [before final aggregation{mean}] of all calculated   m-dimensional Sub-Indices
// vmoutdim         save individual m-k-versions [before final aggregation{mean}] of all calculated   m-dimensional Sub-Indices
// vkout            save individual   m-versions [before final aggregation{mean}] of all calculated   k-dimensional Sub-Indices
// vkoutdim         save individual m-k-versions [before final aggregation{mean}] of all calculated   k-dimensional Sub-Indices
// vmkout           save individual     versions [before final aggregation{mean}] of all calculated m-k-dimensional Sub-Indices
// vmkoutdim        save individual m-k-versions [before final aggregation{mean}] of all calculated m-k-dimensional Sub-Indices
```

Tools

[4.3] Tools for multidimensional weighting | rescaling

a WDim – Design weights | regression for (individual) empirical components

b WDim – Aggregation | Smoothing | Output-Options {Design|Strength|Variation|Correlation}

c Rescale – (Options - Min-Max | Out-Scale)

Programs

Dimwgt - multi-dimensional weighting

ww0 design weights {normative}
wwd dimensional wgts {reg r2_a}
wwi individual wgts {reg predict}
wwf correlation {reg vif}

All dim-wgts | components are normalized {0,…,1}

Default: www [ww0*wwd*wwi*wwf]
Default: wwz [www - smoothed]

```
program define      dimwgt
syntax varlist      [if] [in] [pweight aweight iweight] , idvar(varlist) yvar(varlist)              ///
                    [ covari(varlist) covarc(varlist) hvar(varlist)                                 ///
                    ddesignv(varlist) ddesign(numlist) hdesignv(varlist) hdesign(numlist) wwx(string)        ///
                    smf(real 1000) details dwout dwouthd dwout2 dwoutall dwpre(string) dwsuf(string) minNk(real 100) ]
```

* *** input options
* *** idvar — ids - used to link with frames
* *** pwgt — [individual] weights -- provided as analytical and proportional weights [aw,pw]
* *** yvar — (continuous) — well-being indicators - used as dependent variables in regression [OLS]
* *** covari — (integer) — well-being indicators - used as independent variables in regression [OLS]
* *** covarc — (continuous) — well-being indicators - used as independent variables in regression [OLS]
* *** hvar — (integer) — well-being indicators - used as grouping variable – regression is processed for each group level[k] seperately
* *** xdesign — (vars scores) — external variables with dimensional design weights
* *** design — (scores) — internal dimensional design weights
* *** smf — (value) — smoothing factor [ln(var * smf)] default smf = 1000
* *** minNk — (value) — minimal N [of subgroup-level] to be considered for subgroup estimates
  default minNk = 100
  (for n [of k] < 100 -- dimension estimates without subgroup differentiation are applied)
* *** wwx — (value) — own selection of components [ww0|wwd|wwi|wwf]

Programs

Rescale – Rescaling of variables

```
program define        rescale
syntax varlist        [if] [in] , [vmin(numlist) vmax(numlist) outmin(numlist)  outmax(numlist)  ///
                      vpre(string) vsuf(string) vvn vvo vct0 vct1 ]
```

* *** options

* *** vmin	bottom coding (before rescaling)	
* *** vmax	top coding (before rescaling)	
* *** outmin	assigned minimum value of outvar	
* *** outmax	assigned maximum value of outvar	
* *** vvn	output of normalized outvar	{0,…,1}
* *** vvo	output of designed outvar	{vmin,…,vmax}
* *** vct0	output of standardized outvar	{0,…,100}
* *** vct1	output of standardized outvar	{1,…,100}

Outlook | Discussion

[5] Outlook | Discussion

5.1 Conception – providing structured data for well-being monitoring

5.2 Programs – adjustments | help files - description

5.3 TimeTable – beta-release by September/October – release by December 2024

5.1 Conception – providing structured data for well-being monitoring

Why Multi-dimensional WellBeing Applications?

Identifying individual overlaps in well-being indicators {across conception levels}

Identifying shifts in multiple deprivations | well-beings | inequalities of entire population {for opinion dynamics}

Providing structured (output) indicators for statistical (regression) analyses {individual scores}

Are results from multi-dimensional well-being applications more robust or more volatile [over time] than one-dimensional results?

5.2 Programs – adjustments | help files - description

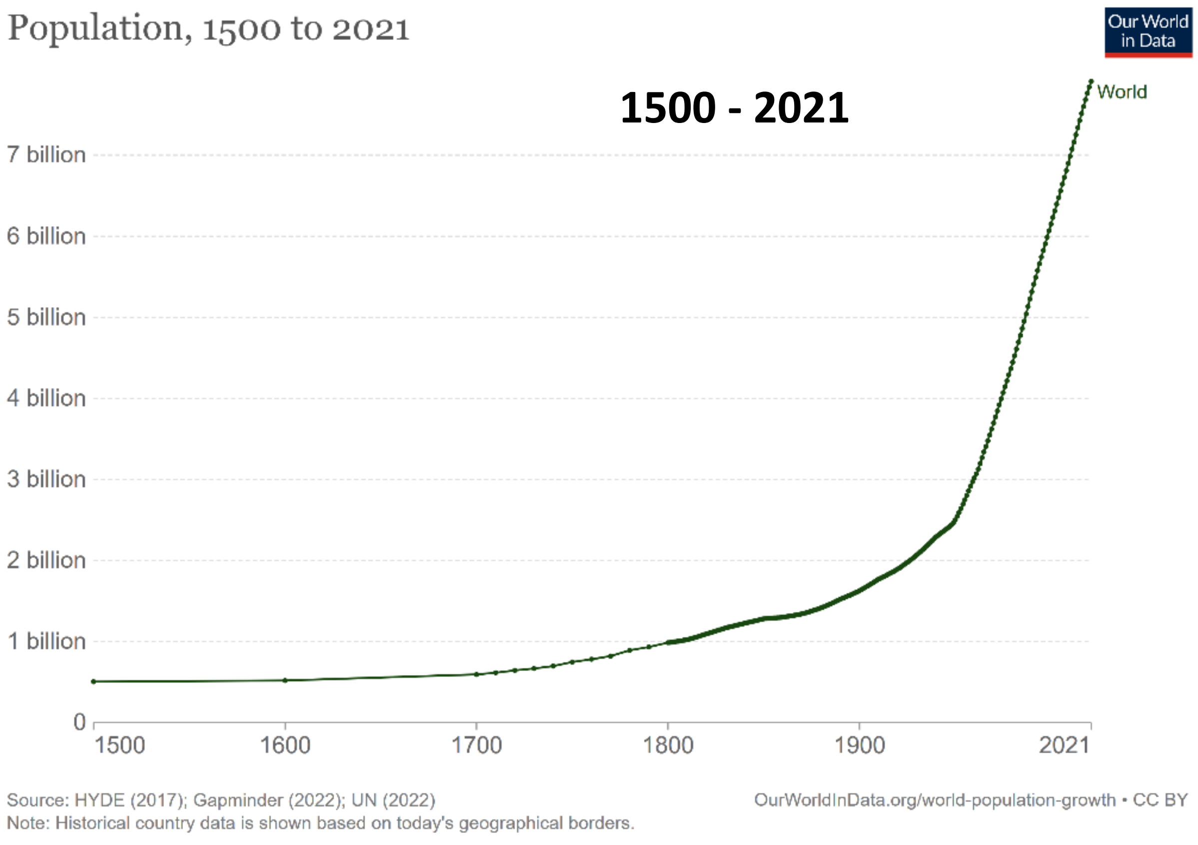
5.3 TimeTable – beta-release by September/October – release by December 2024

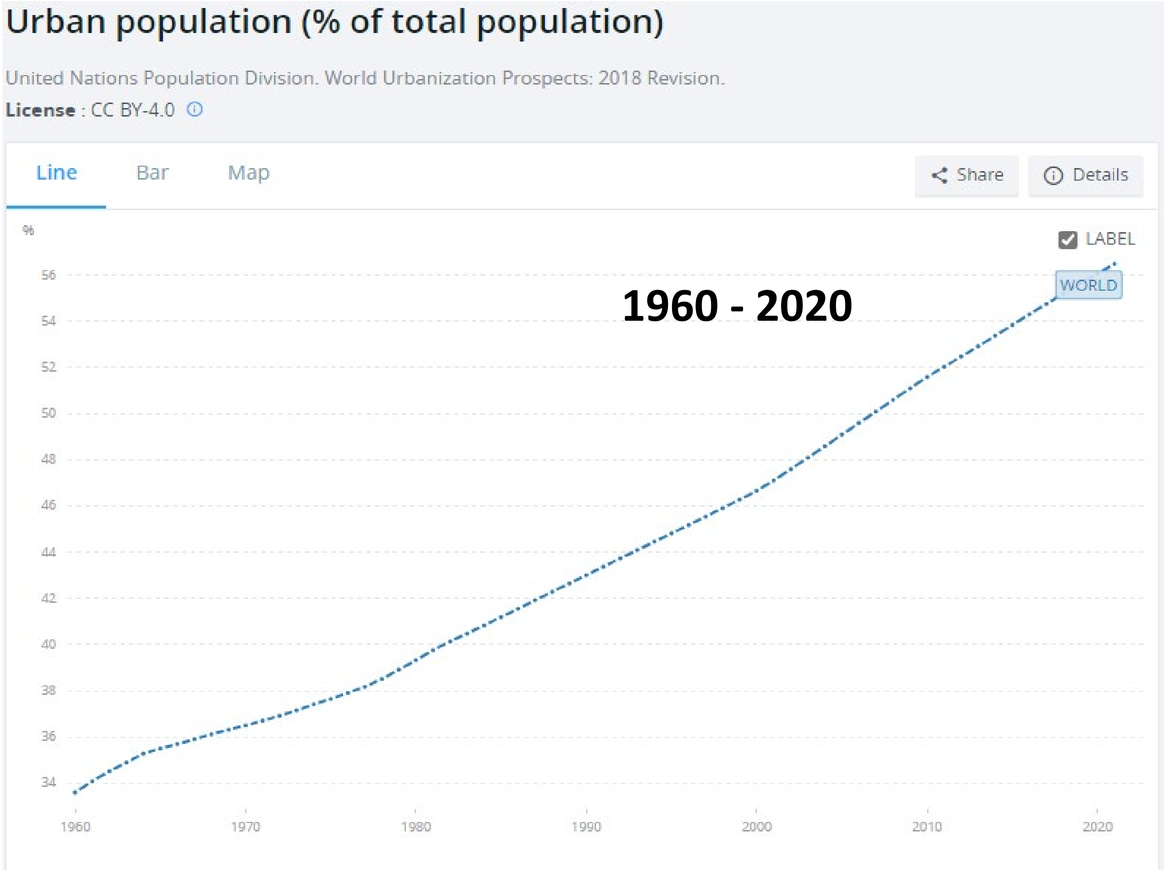
Thank You

Living conditions in urban areas differ from those in rural areas in the much higher density of the population on the one hand and the greater (multi-faceted) [group] diversity on the other.

Urban inequalities are closely related to urban → Subgroup diversities. *[Krause (2023/24) forthcoming]*

Urban Population 2021 % of total population	
World	**56**
France	81
Germany	78
Japan	92
Russian Federation	75
United Kingdom	84
United States	83

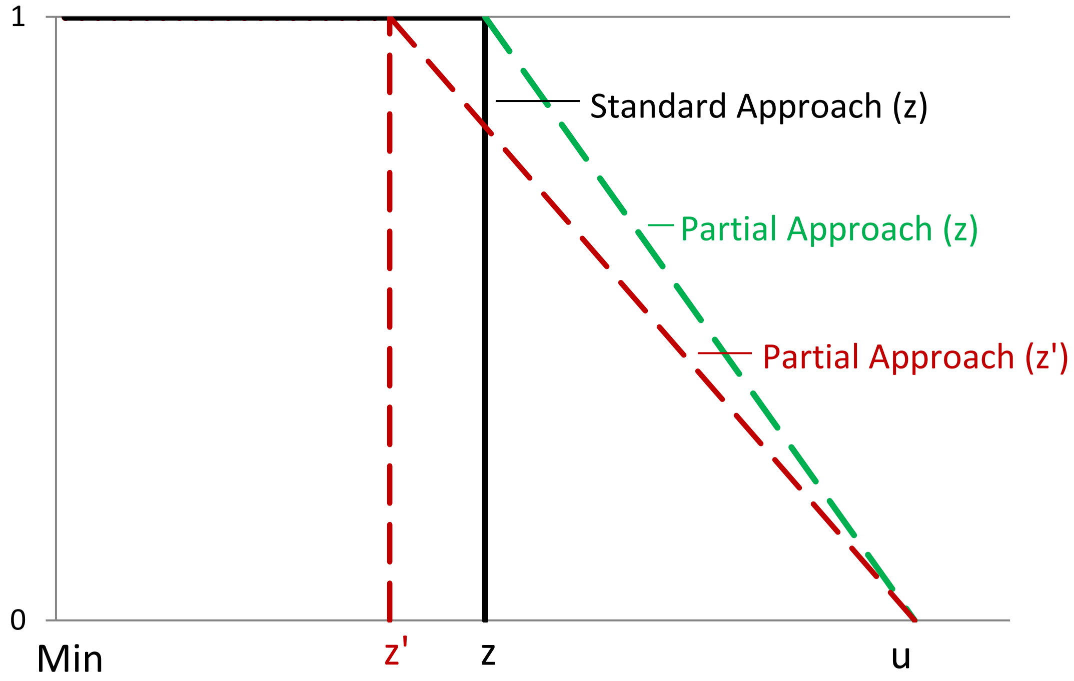




Fuzzy Zones of Precarity

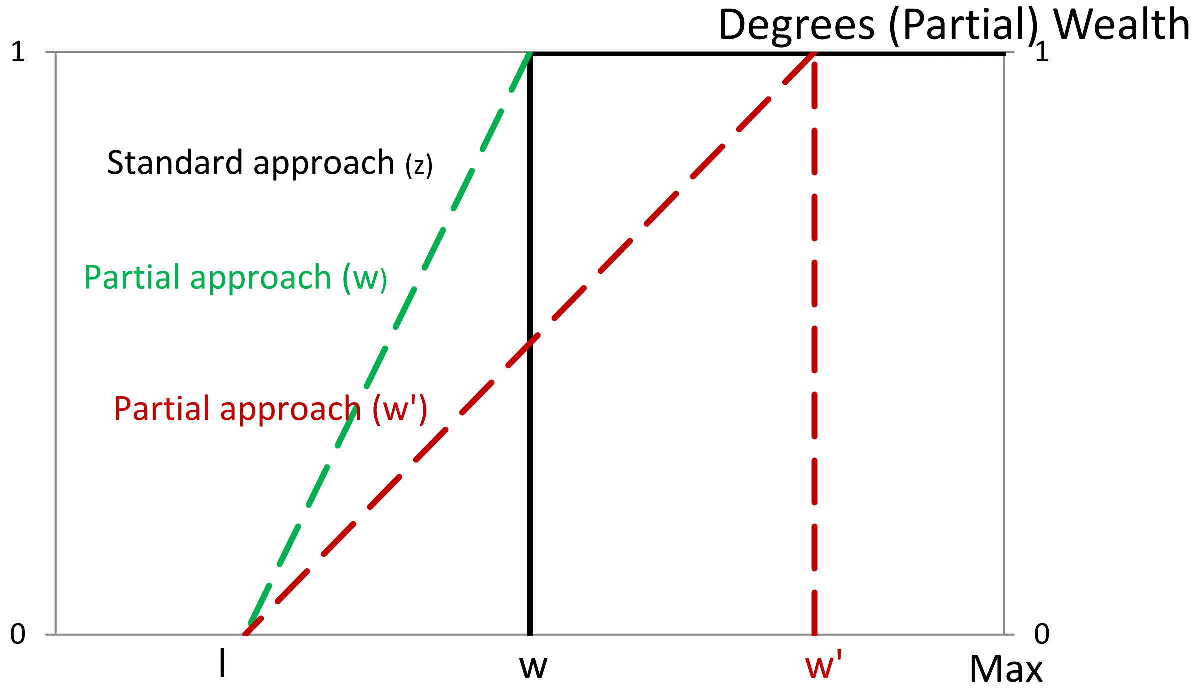
Fuzzy Zones of Prosperity

Fixed-Fuzzy-Approach



- select indicator(s) [dimension]
- define fuzzy zones of precarity [z, u]
- extract degrees of deprivation
- run counting approach

- select indicator(s) [dimension]
- define fuzzy zones of prosperity [w, l]
- extract degrees of wealth
- run counting approach



[Krause (2019; 2023/24) forthcoming]

[Urban] inequalities are closely related to [urban] → Subgroup diversities.

Inequality indicates differences in living conditions as tensions between having and not-having along the (vertical) rich-poor dimension,

whereas **Diversity** reflects (horizontal) between-group variations in ways of living.

Diversities may be characterized – in contrast to horizontal inequalities – as common variations in (multi-dimensional) well-being(s).

Methods – Gini-based Indices

Data and Indicators

[3] Data and Indicators – SOEP | Indicators, Design

3.1 Database SOEP

3.2 Key variables

3.3 Indicators and design

Database German Socio-Economic Panel (SOEP)

For the empirical analyses of urban development, we use the German Socio-Economic Panel (SOEPv37), with its 1,052,596 (continuous) individual observations over the period 1984 to 2020, which is approximately 15,000 to 50,000 annual observations of individuals in private households per year.

The SOEP is a representative longitudinal scientific survey launched in West Germany in 1984 (Wagner, Frick, Schupp 2007). Every year since then, in each SOEP household, a personal interview has been conducted with each individual aged 16 and older. In addition, a household questionnaire is completed by the head of the household or the household member who knows the most about household matters. All households and adult household members are surveyed annually through personal interviews. The main topics covered in the questionnaire are demographics as well as indicators of living conditions, including economic welfare and satisfaction with life (happiness) and different life domains.

Differentiated information is collected on personal and household incomes and further indicators of wealth, housing, labor market issues, health, and leisure (Goebel et al. 2018; Gießelmann et al. 2019). The SOEP was extended to cover the East German population in 1990, so that the transformation of living conditions following unification can be analyzed from the beginning of this process.

SOEPv35, doi:10.5684/soep.v37.

Individual well-being: For *unidimensional analyses*, we concentrate on economic and subjective well-being.

- *Economic well-being*

  Annual household net equivalent incomes (annual previous year incomes at 2020 price levels, adjusted at income-years (t-1); revised OECD scale)

- *Subjective well-being*

  Life satisfaction (0-10 scale, 0 = completely dissatisfied, 10 = completely satisfied)

— both measures are further used as *key variables*
to derive (individual) multi-dimensional weighting schemes

Multidimensional Economic Well-Being

The indicator design defines the reference frame for the analyses; open cells (without indicators) are not considered for interpretation. Cells with several indicators are accordingly down-weighted.

Dimensions	***Sub-dimension***		**Aggregation Levels**			
			intra-personal	personal	social	regional
Econ.Resources	*income*		sat-hinc	pinc	yinc	reg_gdp_pc
					minc_ehc	
	assets				ass_netq	
					ass_groq	

Levels	***Design***	***Dwgt(min)***	***Dwgt(max)***	***SubDimension***	**Indicators**	**Variables**
regional	0.2	.131	.185	reg.income	reg_gdp_pc	Reg.GDP per capita
social [ass.]	0.1	.080	.107	hh.asset	ass_groq	Gross-HH.Assets
	0.1	.079	.106	hh.asset	ass_netq	Net-HH.Assets [>=0]
social [hinc.]	0.1	.098	.128	hh.income	minc_ehc	Monthly eq.Hh-Income EHC
	0.1	.099	.140	hh.income	yinc	Annual eq.Hh-Income
personal	0.2	.164	.231	ind.income	pinc	Personal Annual Income
intra-persona	0.2	.211	.267	sat.income	saty_p	Sat.Hhold-Income

SOEPv37. Design: dimensional design [Sum=1]; Dwgt: dimensional weights [Sum=1];
Household-Income EHC: Household Income after eligible housing costs.

Total population; Satisfaction scores for children are derived from (both) parents|responsible household members.

Generalized well-being applications

First, we distinguish (in the tradition of Maslow, 1946) two types of basic needs dimensions—(daily) life and home—as absolute necessities.

The following three types of economic, social, and cultural well-being resources reflect Pierre Bourdieu’s (1986) capital differentiation on the one hand, and Erik Allardt’s (1993) distinction between having, loving, and being on the other.

Each dimension is further differentiated by subdimensions – for *positive aims* and *negative avoidances* (loss aversion).

Social Reporting: objective and subjective well-being indicators (Glatzer, Zapf 1984)

Social Indicators perspectives for micro, meso, and macro aggregation levels (Noll 2018).

Socio-ecological perspective of interrelated ecological spheres (Bronfenbrenner 1979)

Dimensions	***Sub-dimension***		**(ecological) Aggregation Levels**				
			intra-personal	personal	social	regional	**ecological**
Life	health nutrition recreation *pain*	**D1**					
Home	housing home-production environment *safety*	**D2**					
Econ.Resources	income assets consumption *debts*	**D3**					
Soc.Resources	governance participation quality of life *exclusion*	**D4**					
Cult.Resources	skills tools doings *handicaps*	**D5**					

Individual Multidimensional Weighting Scheme and Rescaling

		Life-Cycle	Source	dweights	sum
1.	Design		{normativ}	**weq_var**	1.00
2.	Living.Std.	{age groups}	ln_yinc		
	Dim.dWeights	{controls}	{r2a_var}	wdinc_var	1.00
	Ind.dWeights		{pre_var/mean_pre_c}	wiinc_var	1.00
3.	Qual.of.Life	{age groups}	sat_life		
	Dim.dWeights	{controls}	{r2a_var}	wdsat_var	1.00
	Ind.dWeights		{pre_var/mean_pre_c}	wisat_var	1.00
4.	Integration &		{weq*wdinc*wiinc*wdsat*wisat}	wdi_raw_var	1.00
	Smoothing		{lnweq*lnwdinc*lnwiinc*lnwdsat*lnwisat*1000}	**wdi_var**	1.00

Table 5 Rescaling of Indicators (for Multidimensional Inequality Applications)

1. Coding	y_c	=	x	
	y_c	=	cmin(x)	if x < cmin(x)
	y_c	=	cmax(x)	if x > cmax(x)
2. Normalization	y_n	=	[y_c – min(y_c)] / [max(y_c) – min(y_c)]	
3. Out-scale (gen.)	y_o	=	y_n * (omax{y_o} – omin{y_o}) + omin(y_o)	
Ct1-scale (1,…,100)	y_ct1	=	y_n * (100 – 1) + 1 .	

Dimension	SubDimension		Indicators	*Mean*	*SD*	*Design*	*Dwgt(mean)*	*Dwgt(min)*	*Dwgt(max)*	Variables
D1.1	**Life**	health	sat_heal	6,6	2,2	0,1000	0,1211	0,1007	0,1466	Sat.Health
D1.2		health	health	3,3	1,0	0,1000	0,1179	0,1023	0,1408	Health Status
D2.1	**Home**	housing	sat_home	7,8	1,9	0,1000	0,1044	0,0890	0,1281	Sat.Home
D2.2		housing	hroom1_pc	1,8	1,0	0,0500	0,0390	0,0343	0,0481	Rooms per capita
D2.3		housing	hroom2_pc	45,8	26,6	0,0500	0,0399	0,0345	0,0484	Area (sqm) per capita
D3.1	**Econ.Res**	income	sat_hinc	6,8	2,1	0,0400	0,0512	0,0417	0,0688	Sat.Hhold-Income
D3.2		income	pinc	34008,9	33420,5	0,0400	0,0403	0,0357	0,0541	Personal Annual Income
D3.3		income	yinc	25696,3	18306,8	0,0200	0,0215	0,0186	0,0295	Annual eq.Hh-Income
D3.4		income	minc_ehc	1409,0	1043,8	0,0200	0,0213	0,0187	0,0289	Monthly eq.Hh-Income EHC
D3.5		assets	ass_netq	134313,3	587812,2	0,0200	0,0162	0,0145	0,0204	Net-HH.Assets [>=0]
D3.6		assets	ass_groq	154615,7	603994,7	0,0200	0,0164	0,0146	0,0205	Gross-HH.Assets
D3.7		income	reg_gdp_pc	37,9	17,0	0,0400	0,0301	0,0239	0,0376	Reg.GDP per capita
D4.1	**Soc.Res**	qual.of.life	sat_life	7,3	1,7	0,0667	0,1017	0,0000	0,1231	Sat.Life
D4.2		participation	pol_int	2,3	0,9	0,0333	0,0265	0,0237	0,0334	Pol.Interest
D4.3		participation	engage	1,1	0,5	0,0333	0,0250	0,0224	0,0303	Engagement
D4.4		participation	cul_int	1,9	0,7	0,0333	0,0304	0,0271	0,0429	Cult.Interest
D4.5		participation	cul_act	1,8	1,0	0,0333	0,0256	0,0222	0,0326	Cult.Engagem.
D5.1	**Cult.Res**	work	lab_emp	7,9	5,5	0,0400	0,0356	0,0274	0,0473	Employment
D5.2		handicap	lab_uem	11,4	2,4	0,0400	0,0348	0,0234	0,0466	UnEmployment
D5.3		work	lab_wi	61,5	44,7	0,0400	0,0355	0,0241	0,0486	HH-WorkIntensity
D5.4		work	reg_emp	59,5	4,3	0,0400	0,0303	0,0245	0,0380	Reg.Empl. per capita
D5.5		skills	educy	12,4	2,8	0,0400	0,0351	0,0316	0,0429	Years-of-Education

SOEPv37. Design: dimensional design [Sum=1]; Dwgt: individual dimensional weights [Sum=1];
Household-Income EHC: Household Income after eligible housing costs.

Results

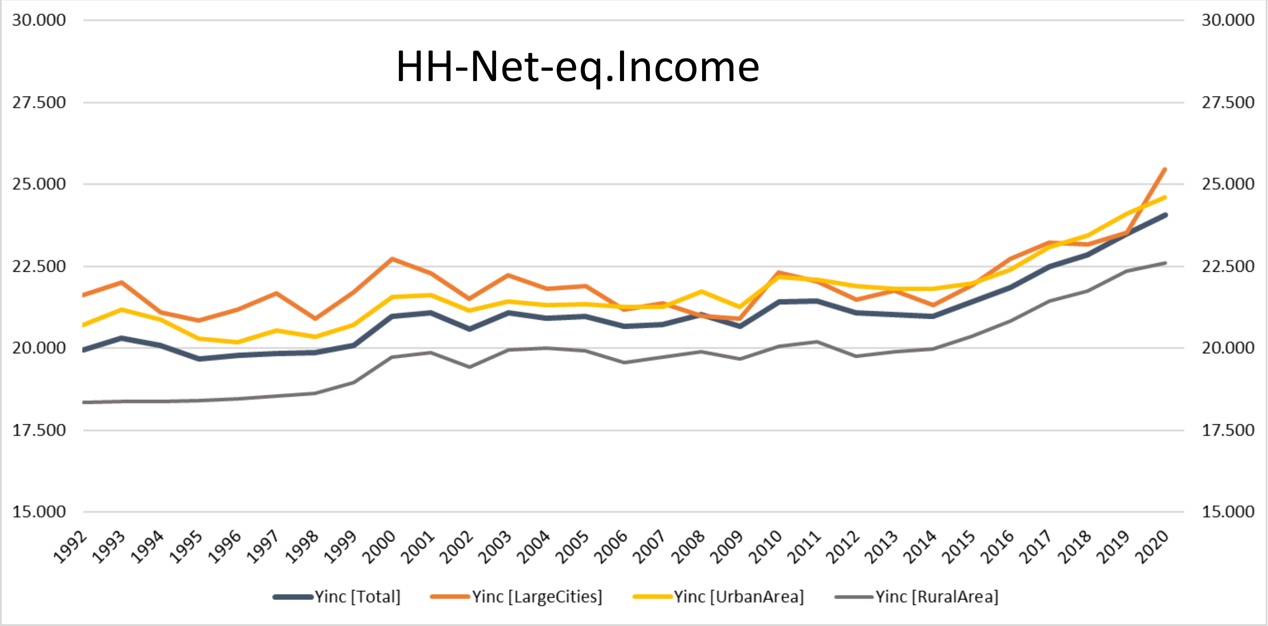
[4] **Results –** Urban Population | Income, Multi-dimensional (Economic) Well-Being

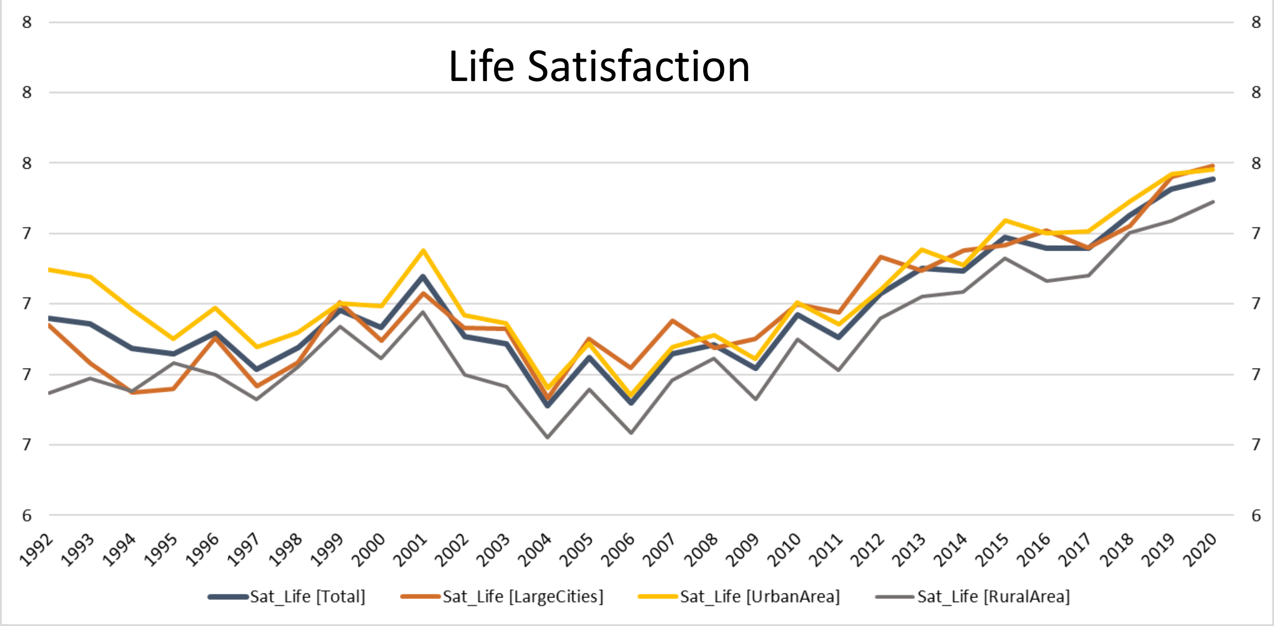
4.1 AF Fixed-fuzzy-approach for deprivation(s) and well-being(s)

4.2 Gini-based inequality(ies) and diversity(ies)

	HH-Net-Income				Life Satisfaction			
1992/94	N-West	S-West	East	*Total*	N-West	S-West	East	*Total*
urban large cities	20688	25413	20779	21619	7,0	7,0	6,4	*6,8*
urban cities	21221	21805	16510	20391	7,1	7,2	6,0	*7,0*
urban small cities	20884	21940	14896	21002	7,2	7,1	5,8	*7,1*
urban area	20708	22217	15834	21296	7,0	7,2	6,1	*7,1*
rural cities	20484	20065	16656	18475	7,1	6,9	6,1	*6,6*
rural area	19288	21161	15818	18303	7,1	7,1	6,2	*6,7*
Total	*20694*	*21893*	*16915*	*20120*	*7,1*	*7,1*	*6,2*	*6,9*
2015/20								
urban large cities	23850	26232	21243	*23316*	7,4	7,5	7,1	*7,3*
urban cities	20997	23596	19477	*21597*	7,3	7,4	7,2	*7,3*
urban small cities	23117	24917	20266	*23743*	7,3	7,4	7,3	*7,4*
urban area	23585	24596	20354	*24018*	7,4	7,4	6,9	*7,4*
rural cities	21427	22276	19466	*20626*	7,4	7,3	6,9	*7,2*
rural area	23059	23310	19757	*22006*	7,4	7,3	7,0	*7,2*
Total	*22681*	*24174*	*20079*	*22673*	*7,4*	*7,4*	*7,0*	*7,3*

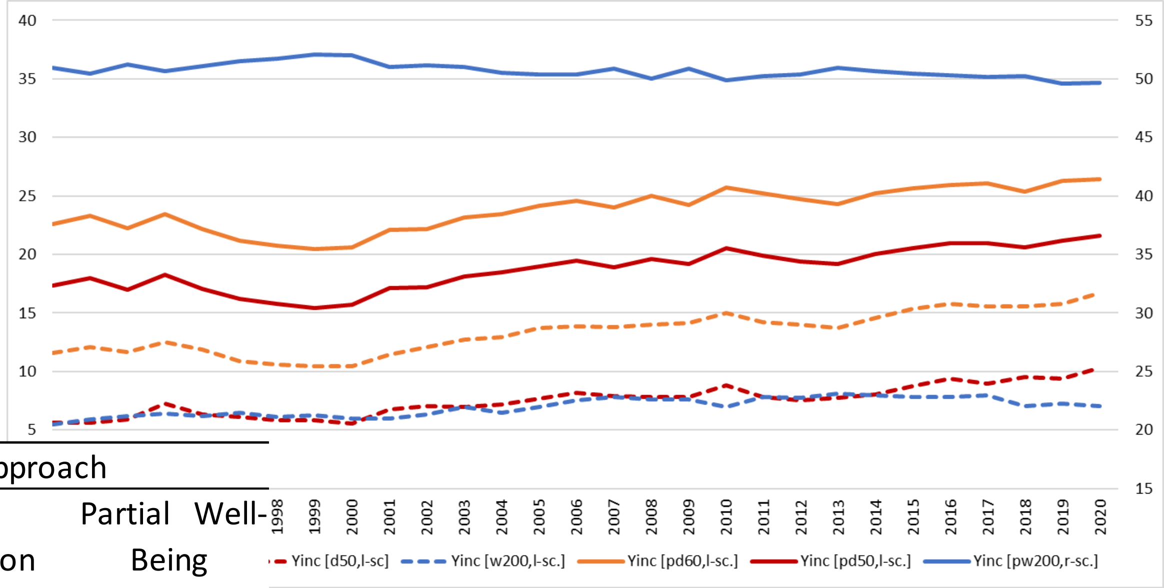
SOEPv37. Med.Annual.Household-Net-Equivalent Incomes [rev.OECD-scale; €-2019, previous year];
Mean.Life satisfaction [0-10-Scale; 0=completely dissatisfied 10=completely satisfied].





Deprivation and Well-Being

HH-Net-Eq.Incomes

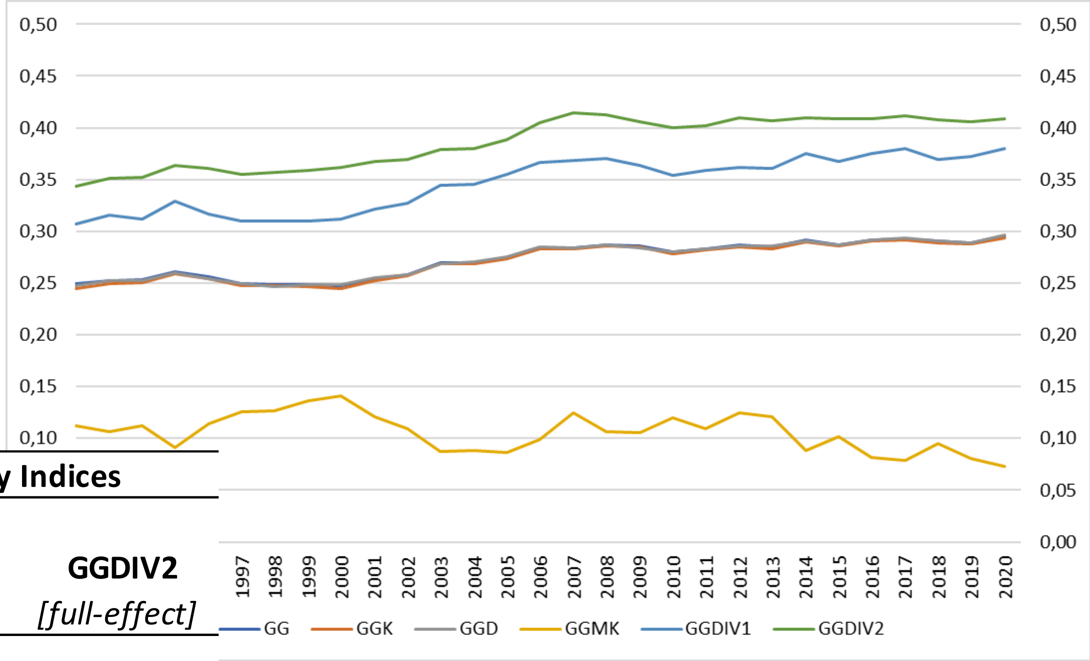


		Standard Approach			Fixed-Fuzzy-Approach		
2020 In Percent	*[in %-med]*	Deprivation *[z60]*	Deprivation *[z50]*	Well-Being *[w200]*	Partial Deprivation *[z60-100]*	Partial Deprivation *[z50-100]*	Partial Well-Being *[w200-50]*
Total		*16,7*	*10,3*	*7,0*	*26,4*	*21,6*	*49,7*
N-West	*(Total)*	*18,1*	*11,7*	*7,5*	*27,5*	*22,6*	*49,1*
	LargeCities	20,0	12,4	12,9	27,0	23,1	53,0
	UrbanArea	17,8	11,7	7,0	27,5	22,4	48,5
	RuralArea	17,2	11,3	4,6	27,8	22,6	47,6
S-West	*(Total)*	*13,9*	*7,7*	*8,2*	*23,0*	*18,6*	*53,7*
	LargeCities	18,3	10,1	14,2	25,9	22,0	56,7
	UrbanArea	13,2	7,8	8,3	22,4	18,3	54,7
	RuralArea	13,7	6,9	5,8	23,0	18,2	50,6
East	*(Total)*	*20,3*	*12,9*	*3,6*	*31,7*	*25,9*	*42,1*
	LargeCities	18,6	12,3	5,6	29,6	24,4	46,5
	UrbanArea	18,2	13,1	2,4	30,2	24,7	43,2
	RuralArea	21,7	13,1	3,1	33,2	27,0	39,6

SOEPv37. Med.Annual.Household-Net-Equivalent Incomes [rev.OECD-scale; €-2019, previous year];
Standard-A. [z=60|50%-med;w=200%-med]; Fuzzy-A.[z,u=60|50-100%med; w,l=200-50%-med].

Inequality and Diversity

HH-Net-Eq.Incomes



2020 Coeff. {0,..,1}		Gini Sub-Indices GG *[main]*	GGK *[within-Gr.]*	GGD *[between-Gr.]*	GGMK *[Gr-diff]*	Diversity Indices GGDIV1 *[net-effect]*	GGDIV2 *[full-effect]*
Total		*0,295*	*0,294*	*0,296*	*0,073*	*0,380*	*0,409*
N-West	*(Total)*	*0,314*	*0,313*	*0,314*	*0,034*	*0,418*	*0,432*
	LargeCities	0,339	0,360	0,334	0,052	0,437	0,458
	UrbanArea	0,314	0,314	0,314	0,031	0,419	0,432
	RuralArea	0,292	0,270	0,299	0,028	0,400	0,412
S-West	*(Total)*	*0,285*	*0,283*	*0,289*	*0,101*	*0,362*	*0,401*
	LargeCities	0,349	0,403	0,342	0,133	0,439	0,485
	UrbanArea	0,283	0,280	0,287	0,094	0,361	0,398
	RuralArea	0,267	0,247	0,276	0,105	0,337	0,378
East	*(Total)*	*0,267*	*0,266*	*0,273*	*0,084*	*0,348*	*0,380*
	LargeCities	0,291	0,311	0,283	0,119	0,353	0,398
	UrbanArea	0,262	0,256	0,263	0,051	0,341	0,363
	RuralArea	0,258	0,247	0,271	0,077	0,348	0,377

SOEPv37. Med.Annual.Household-Net-Equivalent Incomes [rev.OECD-scale; €-2019, previous year];
GG… population adjusted Gini Sub-Indices.

Inequality and Diversity HH-Net-Eq.Incomes

2020	GG	GK	GD	GMK	GDD	GDQ	GDDR	GDQR	GDQM
					Gini Sub-Indices				
N-West	0,314	0,136	0,178	0,020	0,197	0,039	0,210	0,034	0,019
S-West	0,285	0,127	0,158	0,057	0,150	0,048	0,186	0,033	0,052
East	0,267	0,108	0,159	0,050	0,156	0,047	0,188	0,034	0,045

Gini Sub-Indices [GG… population adjusted]

2020	GG	GGK	GGD	GGMK	GGDD	GGDQ	GGDDR	GGDQR	GGDQM
N-West	0,314	0,313	0,314	0,034	0,349	0,069	0,372	0,060	0,032
S-West	0,285	0,283	0,289	0,101	0,275	0,087	0,341	0,060	0,093
East	0,267	0,266	0,273	0,084	0,268	0,080	0,323	0,058	0,076

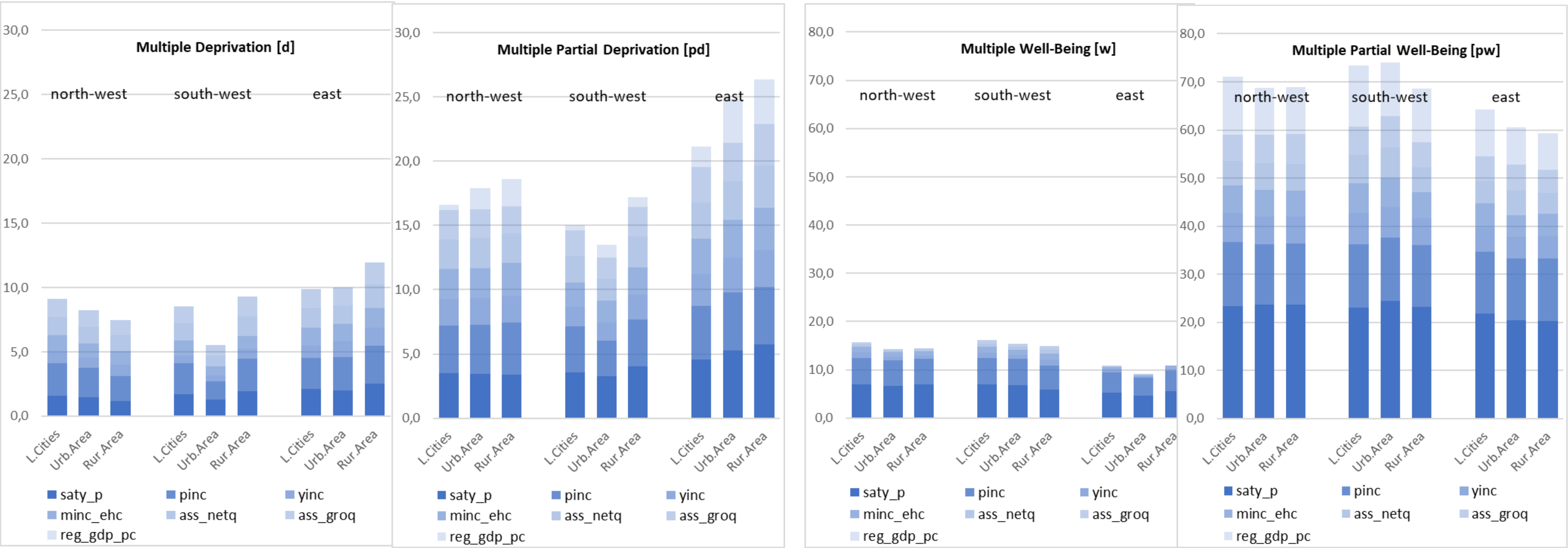
Diversity indices [GG… population adjusted]

2020	GDIV1	GDIV2	GGDIV1	GGDIV2	*Subgroup - Levels Urban Indicator*
N-West	0,236	0,244	0,418	0,432	*LargeCities*
S-West	0,198	0,219	0,362	0,401	*UrbanArea*
East	0,203	0,222	0,348	0,380	*RuralArea*

SOEPv37. Annual.Household-Net-Equivalent Incomes [rev.OECD-scale; €-2019, previous year].

GG… population adjusted Gini Sub-Indices.

M-Dim Economic Resources – Deprivation and Well-Being [standard and fixed-fuzzy approach, AF]

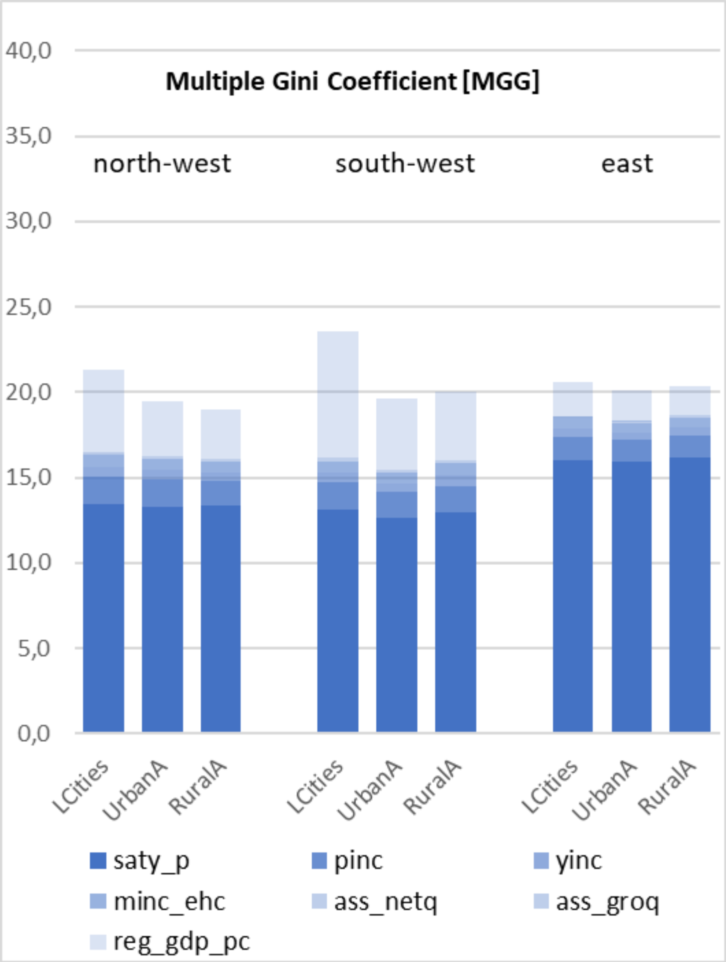


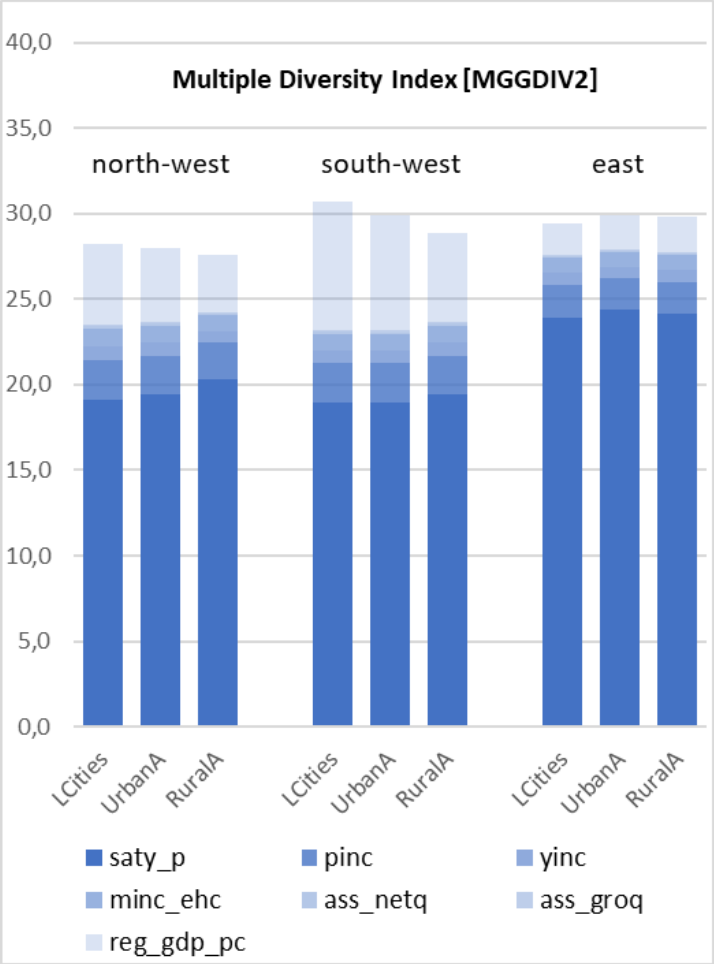
SOEPv37 [2017;
total population]

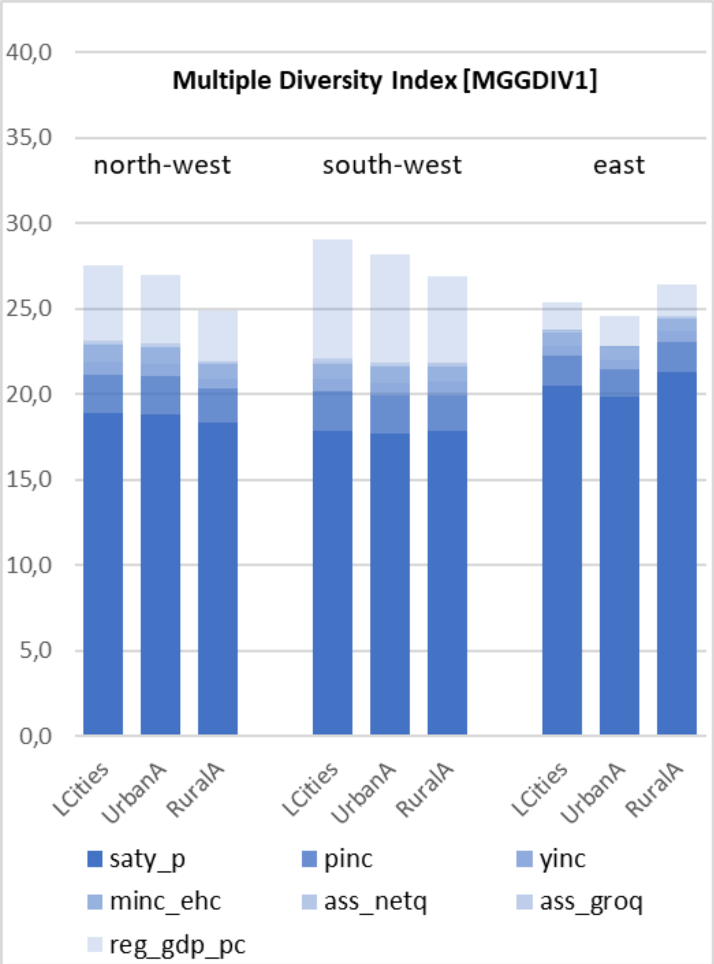
Levels	*Design*	*Dwgt(min)*	*Dwgt(max)*	*SubDimensioı*	Indicators	Variables
regional	0.2	.131	.185	reg.income	reg_gdp_pc	Reg.GDP per capita
social [ass.]	0.1	.080	.107	hh.asset	ass_groq	Gross-HH.Assets
	0.1	.079	.106	hh.asset	ass_netq	Net-HH.Assets [>=0]
social [hinc.]	0.1	.098	.128	hh.income	minc_ehc	Monthly eq.Hh-Income EHC
	0.1	.099	.140	hh.income	yinc	Annual eq.Hh-Income
personal	0.2	.164	.231	ind.income	pinc	Personal Annual Income
intra-persona	0.2	.211	.267	sat.income	saty_p	Sat.Hhold-Income

SOEPv37. Design: dimensional design [Sum=1]; Dwgt: dimensional weights [Sum=1];
Household-Income EHC: Household Income after eligible housing costs.

M-Dim Economic Resources – inequalities and diversities [Gini-based sub-indices]





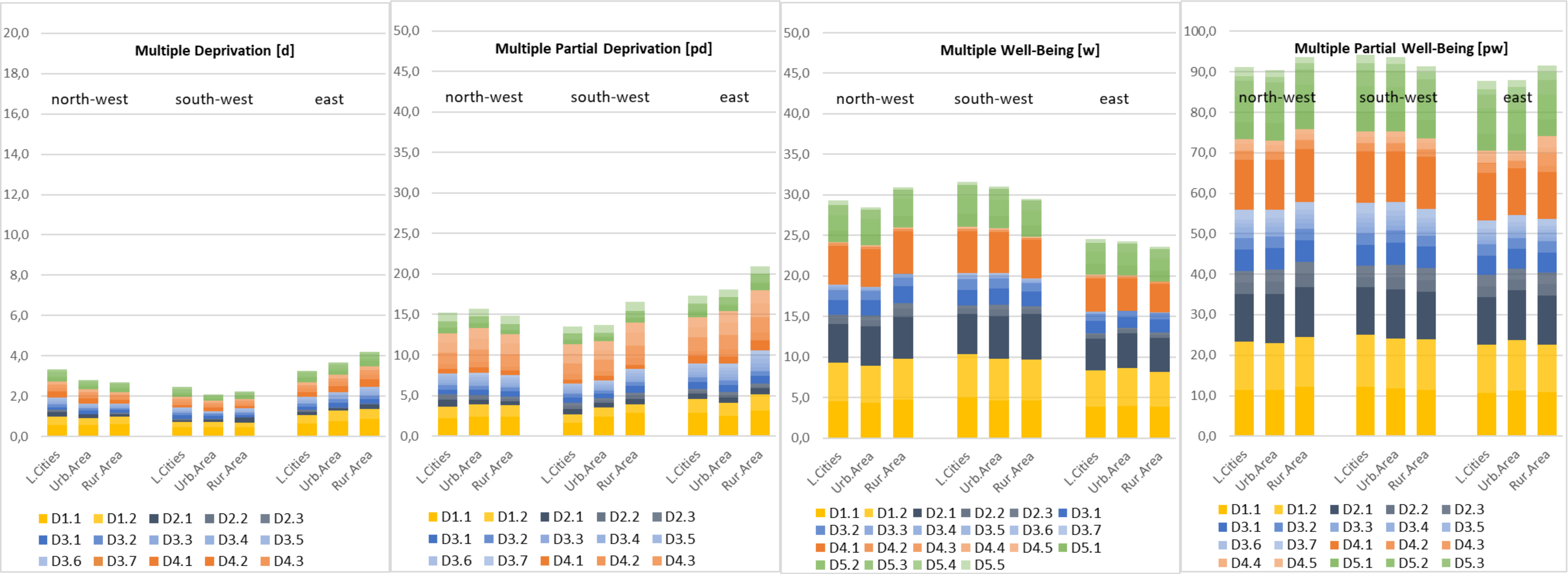


SOEPv37 [2017;
total population]

2017	MGG	MGGK	MGGD	MGGMK	MGGDIV1	MGGDIV2
N-West	0,198	0,191	0,208	0,052	0,269	0,280
S-West	0,203	0,195	0,226	0,071	0,283	0,300
East	0,203	0,198	0,206	0,115	0,252	0,297

SOEPv37. Subgroups [within- and between-groups]: Large Cities, Urban Areas, Rural Areas

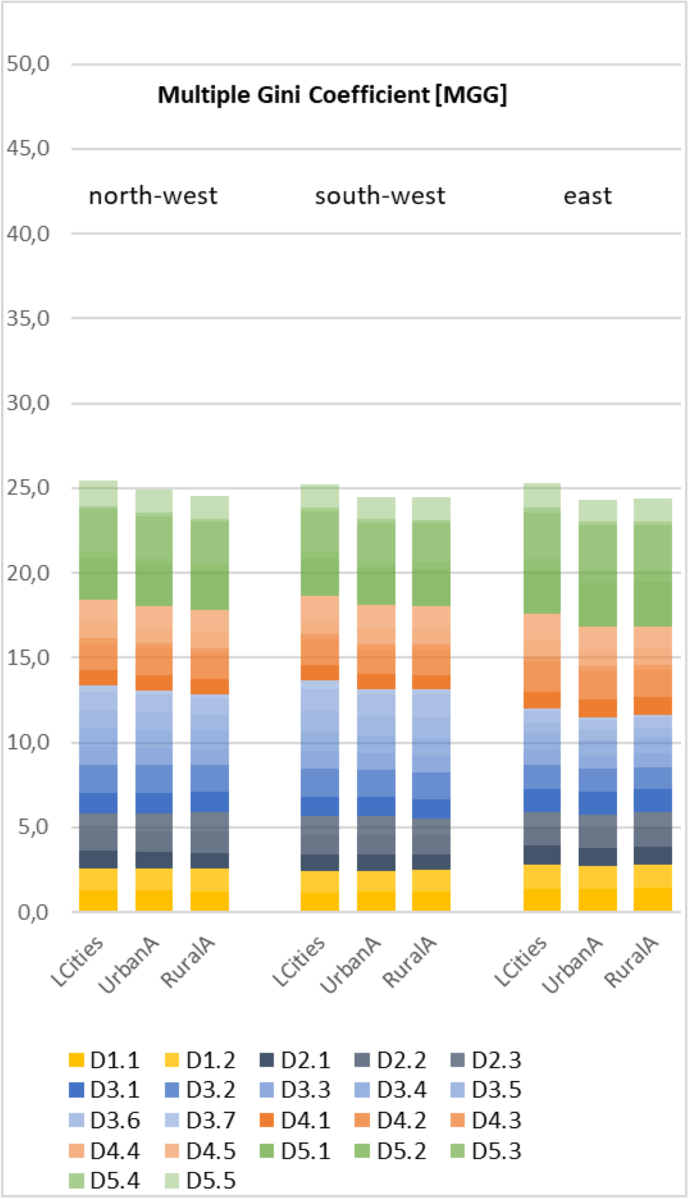
M-Dim General Resources – Deprivation and Well-Being [standard and fixed-fuzzy approach]

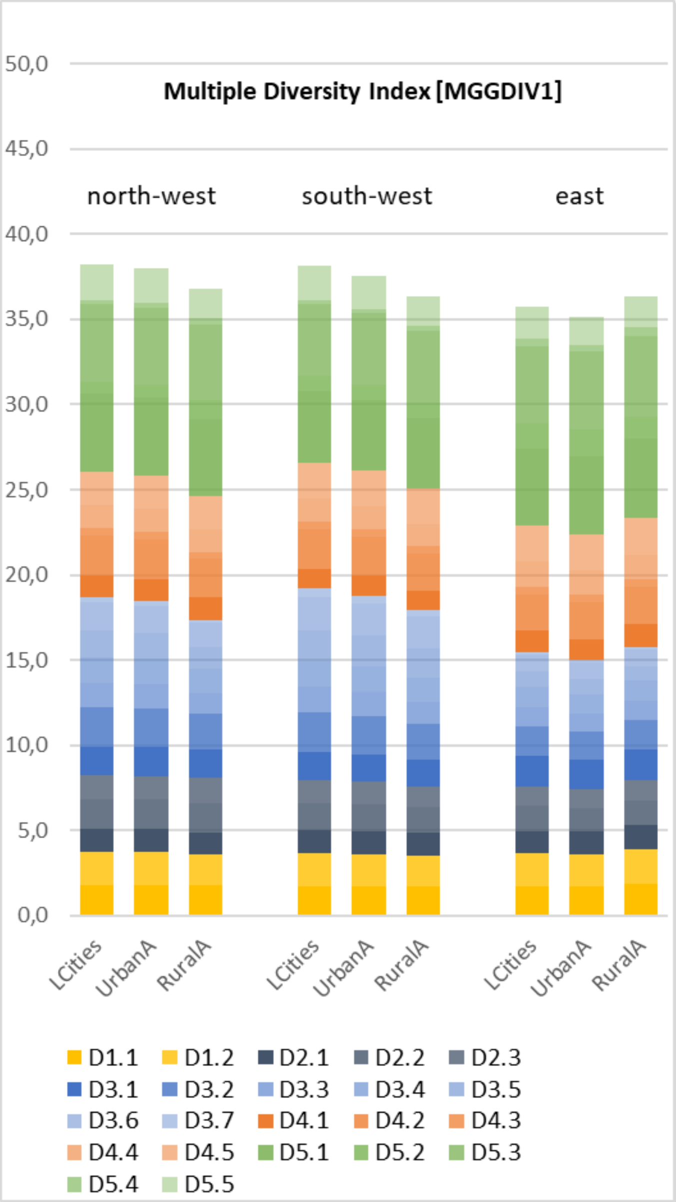


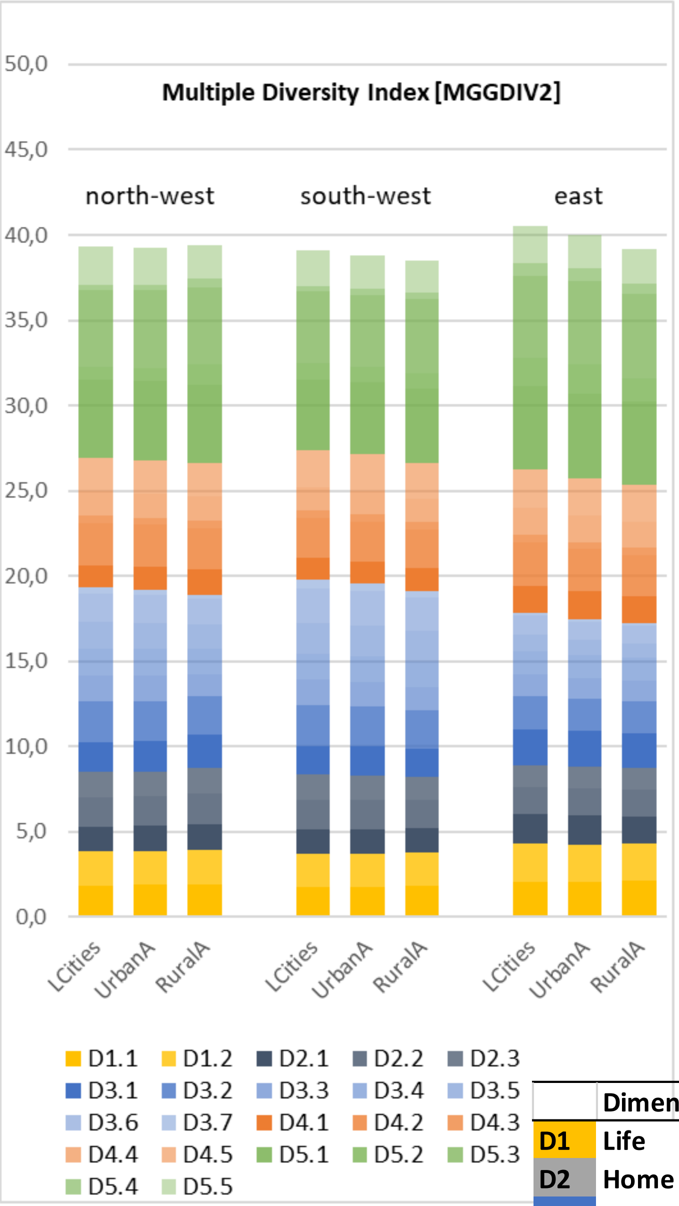
SOEPv37 [2017;
adult population]

	Dimension	SubDimension
D1	**Life**	health
D2	**Home**	housing
D3	**Econ.Resources**	income, assets
D4	**Soc.Resources**	qual.of.life, participation
D5	**Cult.Resources**	work,skills,handicap

M-Dim General Resources – inequalities and diversities [Gini-based sub-indices]







SOEPv37 [2017;
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	Dimension	SubDimension
D1	**Life**	health
D2	**Home**	housing
D3	**Econ.Resources**	income, assets
D4	**Soc.Resources**	qual.of.life, participation
D5	**Cult.Resources**	work,skills,handicap

Discussion

[5] **Discussion –** Emergence of urban inequalities and diversities in Germany

5.1 Urban Inequalities and Diversities

Empirical results on urban-rural inequalities in Germany

In summary, our results confirm several regional differences, especially when considering deprivation and economic well-being in East Germany, but show almost no systematic variations in inequality or diversity in our urban categories—large cities, other urban areas, and rural areas—in and across regions. Several Gini-based subindices indicate instead, for all unidimensional and multidimensional settings, high diversity-related overlaps in distributions, with similar pictures of within- and between-group variation and low mean group differences. Living conditions in Germany are thus not really divided by differences between large cities, other urban areas, and rural areas.

Instead, we observe higher degrees of inequality over time—also within each kind of urban and rural area: Large cities display substantial internal variation between rich and poor parts of the city; economic levels differ between urban and suburban areas; and rural towns and municipalities are very heterogenous (economically).

Gini-based Diversities …

- … need further (empirical and methodological) checks
- … re-adress [between-group] inequalities as joint variations across social groups
- … point to our notion as group members in social interactions
- … and may be related to our collective capabilities to manage our (collective) identities and (social) roles

Thank You

Methods

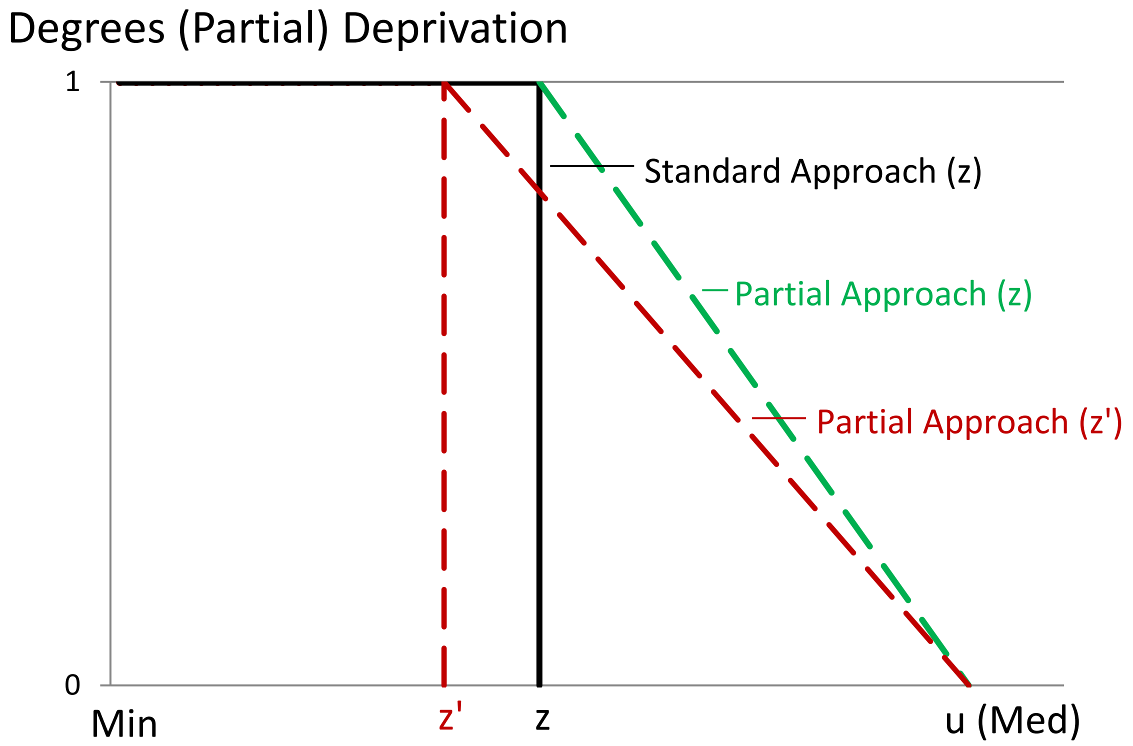
Methods - Partial Deprivation and Partial Wealth

- ❖ Identification of (Partial) Deprivation and Wealth
- ❖ Aggregation of (Partial) Deprivation and Wealth

Following Sen (1976) we face two general steps in the measurement of deprivation … . First we have to choose indicators and to set up thresholds to identify someone as poor – the **identification step**. Second we have to set up measures to account for the incidence, intensity and inequality of poverty – the **aggregation step**.

This presentation offers a counting approach for the identification of **partial deprivation** and **partial wealth –**
… and it‘s aggregation according to the well-established FGT-Measures, as proposed by Foster/Greer/Thorbecke (1984; 2010)

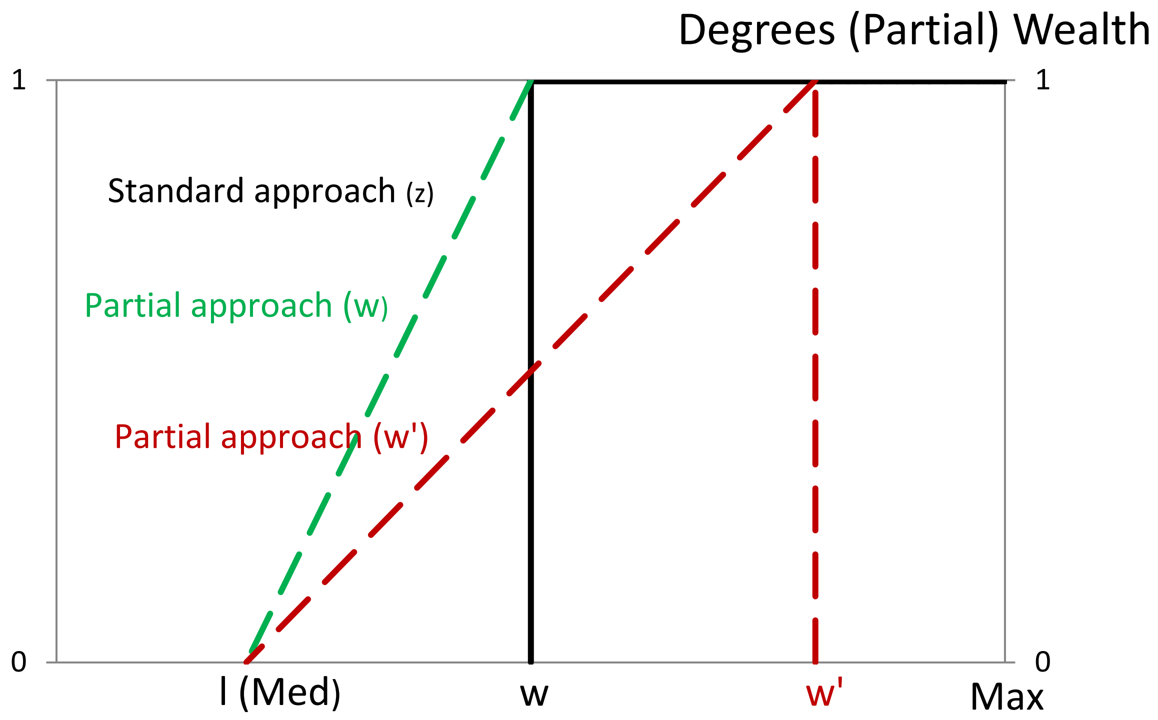
Fuzzy Zones of Precarity



- select indicator(s) [dimension]
- define fuzzy zones of precarity [z, u]
- extract degrees of deprivation
- run counting approach

Fuzzy Zones of Prosperity

- select indicator(s) [dimension]
- define fuzzy zones of prosperity [w, l]
- extract degrees of wealth
- run counting approach



Identification of Partial Deprivation
Identification of Partial Wealth

[1] Identification of Poverty

$$hz_i = 1 \text{ if } y_i < z$$

$$hz_i = 0 \text{ if } y_i \geq z$$

with

y_i individual indicator for poverty
z poverty threshold
hz_i individual poverty score [0, 1]

[1] Identification of Wealth

$$hw_i = 1 \text{ if } x_i > w$$

$$hw_i = 0 \text{ if } x_i \leq w$$

with

x_i individual indicator for wealth
w wealth threshold
hw_i individual wealth score [0, 1]

[2] Identification of Partial Deprivation

$$pd_i = exp\left\{\varepsilon\left[1-\left(\frac{y_i}{z}\right)^{\tau\left[\left(\frac{y_i}{z}\right)\right]}\right]\right\}$$

with

pd_i individual score of partial deprivation for indicator y_i

y_i individual value of deprivation in indicator y

z threshold of poverty/deprivation for indicator y

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

[2] Identification of Partial Wealth

$$pw_i = exp\left\{\varepsilon\left[1-\left(\frac{x_i}{w}\right)^{-\tau\left[\left(\frac{x_i}{w}\right)^{-1}\right]}\right]\right\}$$

with

pw_i individual score of partial wealth for indicator x_i

x_i individual value of wealth in indicator x

w threshold of wealth for indicator x

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function

[2a] Identification of Partial Deprivation

Defining the "zone of precarity" (b)

$$b = \frac{u}{z}$$

with

z poverty/deprivation threshold
u upper limit for partial deprivation

[2a] Identification of Partial Wealth

Defining the "zone of prosperity" (q)

$$q = \frac{l}{w}$$

with

w wealth threshold
l lower limit for partial wealth

[2b] Identification of Partial Deprivation

Defining [lim] for the "zone of precarity" (b)

$$\lim\_u = \; exp\{\varepsilon[1 - b^{\tau[b]}]\}$$

with

$\lim\_u$ marginal rate of partial deprivation at the upper limit (u) [default: lim_u = .01]

b ratio of lower to upper limits (u / z) for the fuzzy zone of deprivation

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function [default: ε = 1.0]

[2b] Identification of Partial Wealth

Defining [lim] for the "zone of prosperity" (q)

$$\lim\_l = \; exp\{\varepsilon[1 - q^{-\tau[q^{-1}]}]\}$$

with

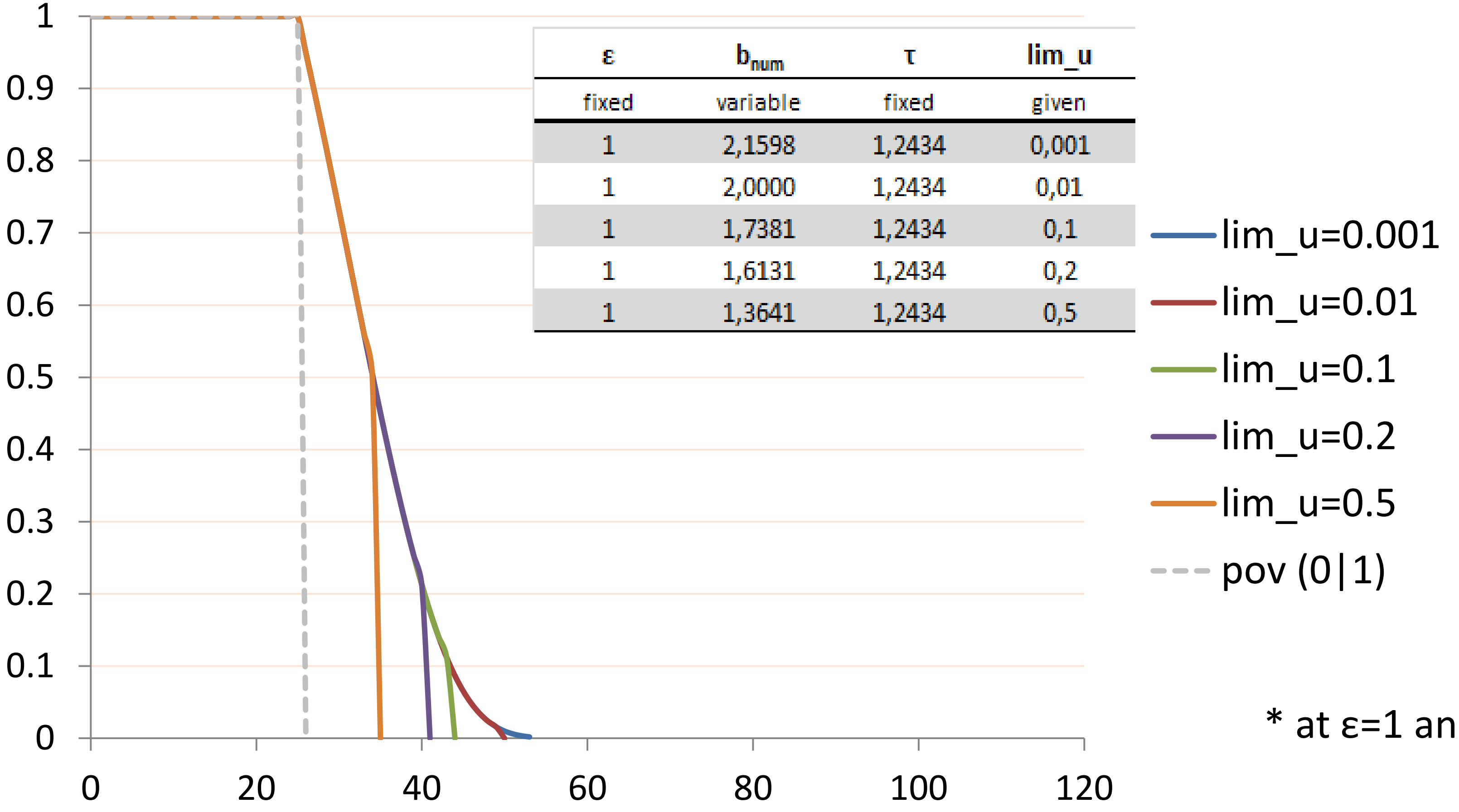
$\lim\_l$ marginal rate of partial wealth at the lower limit (l) [default: lim_ l = .01]

q ratio of lower to upper limits (l / w) for the fuzzy zone of prosperity

τ parameter for the type of the baseline identification function

ε parameter for the shape of the identification function [default: ε = 1.0]

Different lim_u* → Segmentation Approach



ε	b_{num}	τ	lim_u
fixed	variable	fixed	given
1	2,1598	1,2434	0,001
1	2,0000	1,2434	0,01
1	1,7381	1,2434	0,1
1	1,6131	1,2434	0,2
1	1,3641	1,2434	0,5

* at ε=1 and τ=1,2434

[2c] Identification of Partial Deprivation

Setting the baseline function (τ) for partial deprivation at the "zone of precarity":

$$\tau = -\frac{\ln\left[1 - \frac{\ln(\lim\_u)}{\varepsilon}\right]}{b\ln(b)}$$

Modelling the shape of the slope (ε) for the baseline function:

$$\varepsilon = \frac{\ln(\lim\_u)}{1 - b^{\tau[b]}}$$

[2c] Identification of Partial Wealth

Setting the baseline function (τ) for partial wealth at the "zone of prosperity":

$$\tau = -\frac{\ln\left[1 - \frac{\ln(\lim\_l)}{\varepsilon}\right]}{q^{-1}\ln(q)}$$

Modelling the shape of the slope (ε) for the baseline function:

$$\varepsilon = \frac{\ln(\lim\_l)}{1 - q^{-\tau[q^{-1}]}}$$

[3] Identification of Poverty *and* Partial Deprivation

$$hz_i = 1 \text{ if } y_i < z$$
$$hz_i = pd_i \text{ if } u > y_i \geq z$$
$$hz_i = 0 \text{ if } y_i \geq u$$

with

y_i individual indicator for poverty
z poverty threshold (lower limit for fuzzy zone of precarity)
u precarity threshold (upper limit for fuzzy zone of precarity)
hz_i individual poverty score [0,1]
pd_i individual partial deprivation score within the fuzzy zone of precarity [0,…,1]

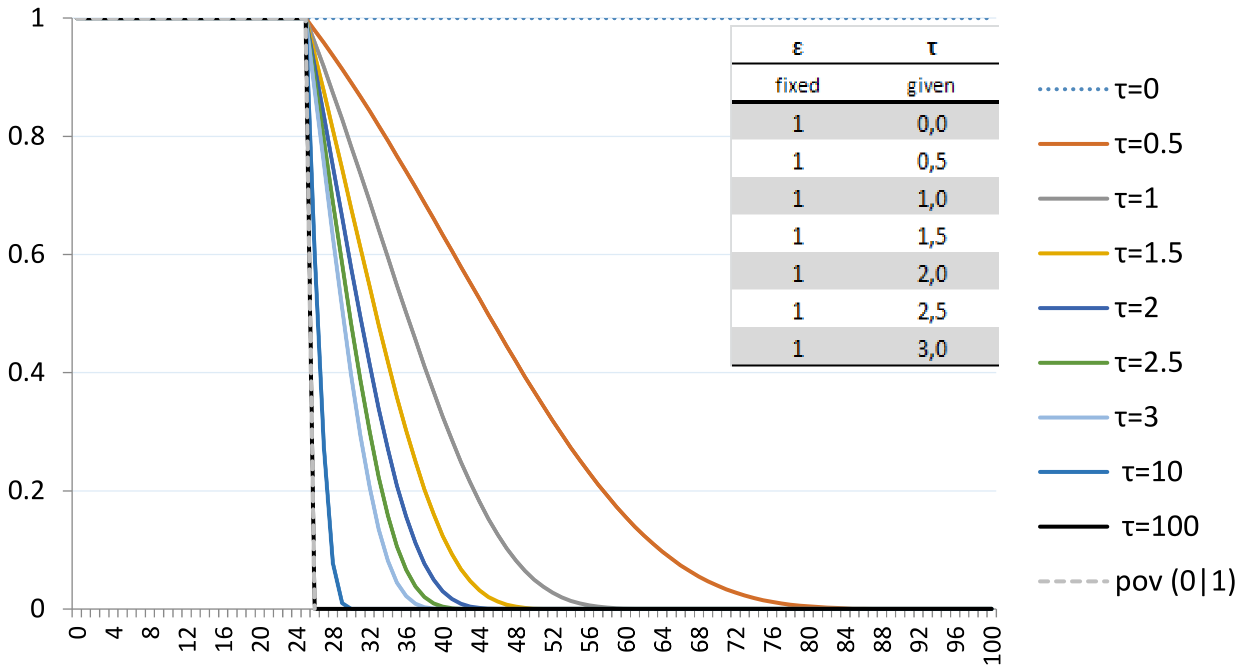
[3] Identification of Wealth *and* Partial Wealth

$$hw_i = 1 \text{ if } x_i \geq w$$
$$hw_i = pw_i \text{ if } w > x_i \geq l$$
$$hw_i = 0 \text{ if } x_i \leq l$$

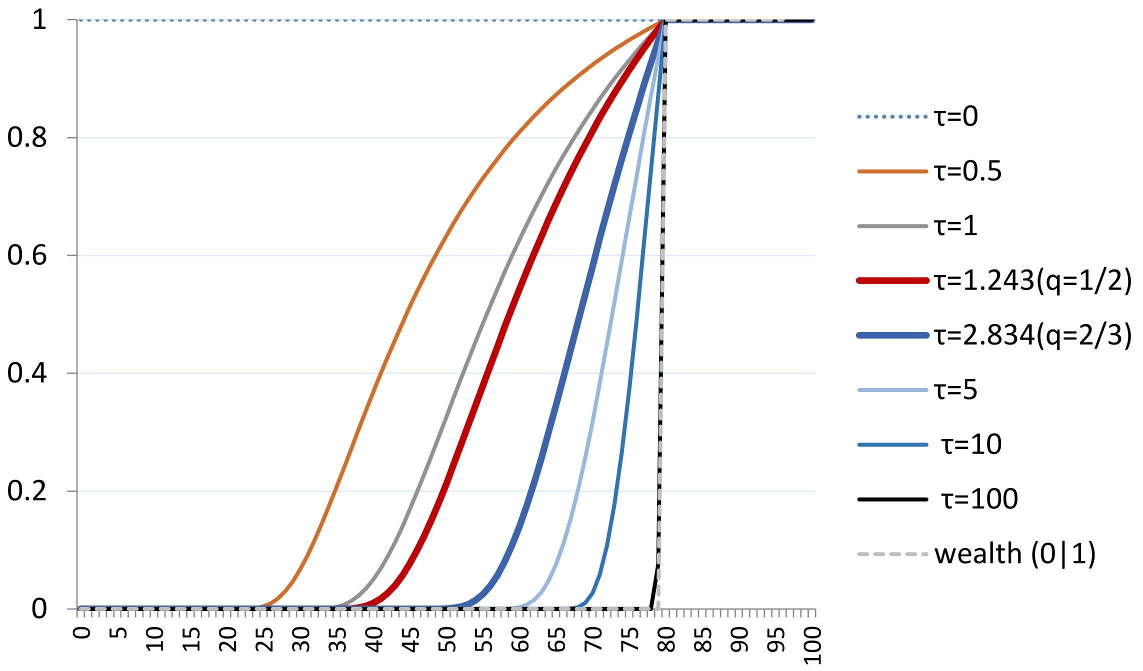
with

x_i individual indicator for wealth
w wealth threshold (upper limit for fuzzy zone of prosperity)
l prosperity threshold (lower limit for fuzzy zone of prosperity)
hw_i individual wealth score [0,1]
pw_i individual partial wealth score within the fuzzy zone of prosperity [0,…,1]

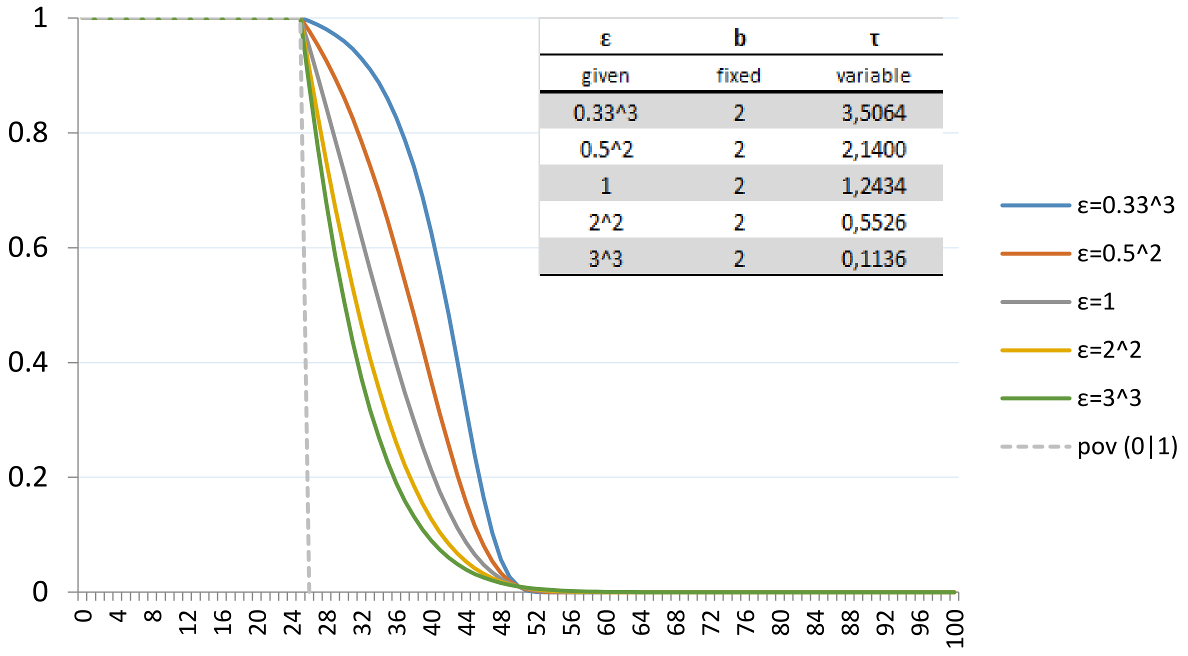
[3a] Identification of Poverty *and* Partial Deprivation *baseline function (τ)*



[3a] Identification of Wealth *and* Partial Wealth *baseline function (τ)*

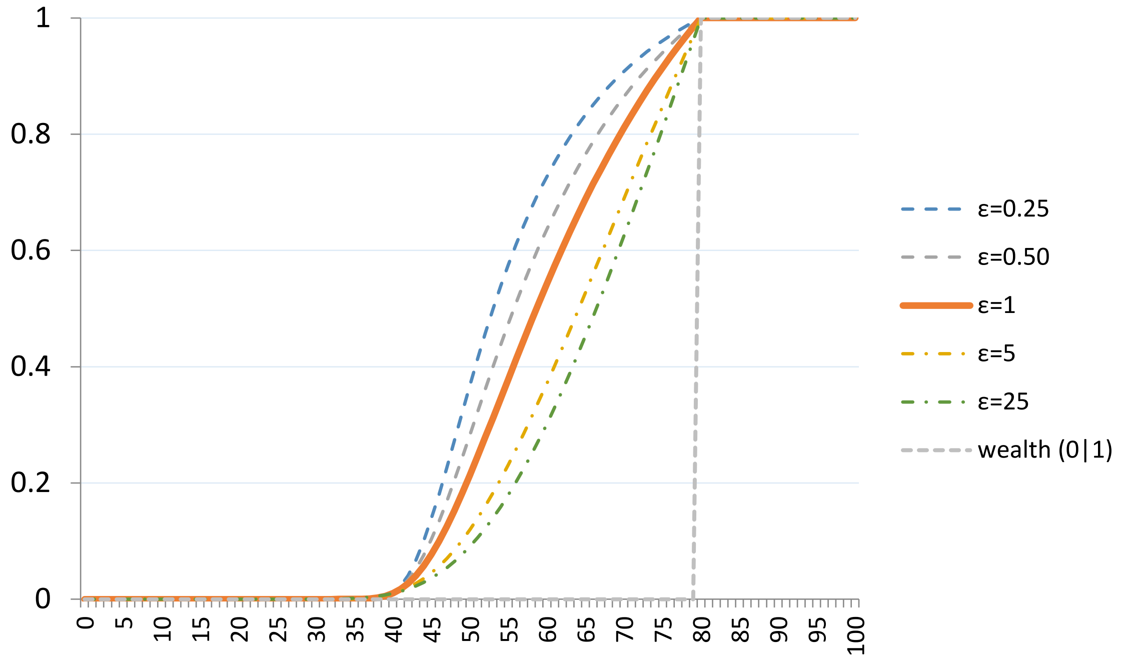


[3b] Identification of Poverty *and* Partial Deprivation <u>*shape of the slope (ε)*</u>

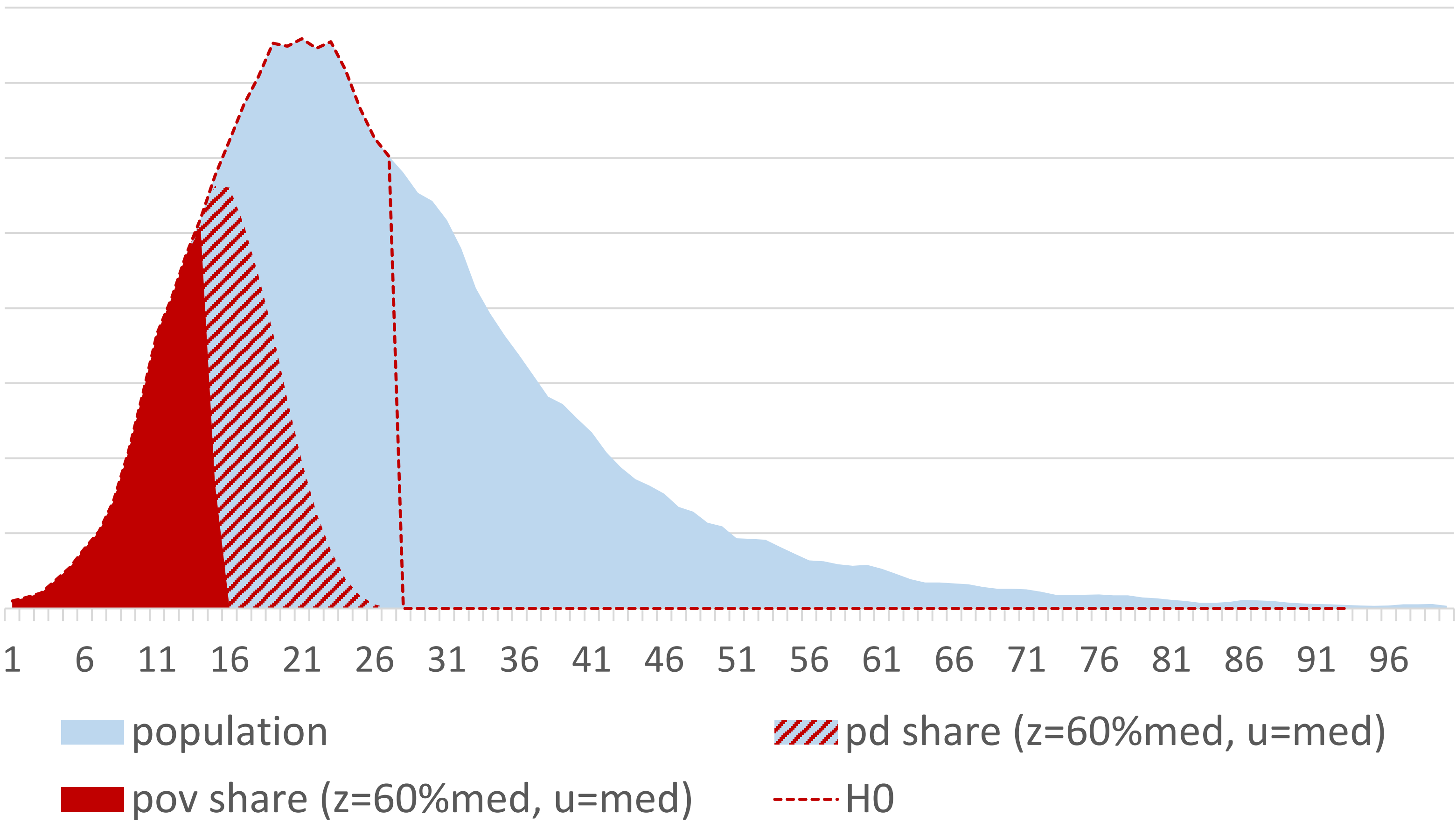


ε	b	τ
given	fixed	variable
0.33^3	2	3,5064
0.5^2	2	2,1400
1	2	1,2434
2^2	2	0,5526
3^3	2	0,1136

[3b] Identification of Wealth *and* Partial Wealth <u>*shape of the slope (ε)*</u>



Distribution of conventional poverty rates (60%med) and partial deprivation rates in Germany, 2009-2013



SOEPv31l (2010-2013); z=60%med. pd: τ=2.0246, b=5/3, u=med, lim_u=0.01.

Aggregation of Partial Deprivation
Aggregation of Partial Wealth

[4] FGT-Measurement of Poverty

In 1984 Foster, Greer, and Thorbecke presented their now well-established FGT measures of poverty:

$$FGT = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{z - y_i}{z} \right)^{\alpha}$$

[4a] Modified FGT-Measurement of Poverty

for absolute and relative specifications of gaps:

$$FGTmod = \frac{1}{n} \sum_{i=1}^{q} \left(\frac{z - y_i}{z - dmin} \right)^{\alpha}$$

[5] Generalized FGT Measurement of Poverty

The equation for the FGT measure of poverty can be generalized for the usual standard measures of dichotomous poverty h_i {1,0}, with (1 = poor) and (0 = non-poor):

$$h_i = \{0,1\}$$

$$i_i = \left(\frac{z - y_i}{z}\right)^{\alpha} \text{ for } y_i < z$$

$$FGT = \frac{1}{n} \sum_{i=1}^{n} h_i \, i_i$$

[6] Adjusted FGT Aggregation of Poverty

$$y_i^{\ *} = \frac{z - y_i}{z - d_{min}} \qquad y_i^{\ *} \in [0, \dots, 1]$$

$$hz_i = \{0,1\} \quad \text{identification: } hz_i = 1 \text{ for } y_i < z$$

$$iz_i = iz(z)_i = \left(\frac{z - y_i}{z - d_{min}}\right)^{\alpha} \text{ for } y_i \leq z$$

$$zFGT = \frac{1}{n} \sum_{j=1}^{n} hz_i \, iz_i$$

[6] Adjusted FGT Aggregation of Wealth

$$x_i^{\ *} = \frac{x_i - \mathrm{w}}{w_{max} - \mathrm{w}} \qquad x_i^{\ *} \in [0, \dots, 1]$$

The proposed FGT measure for wealth can such be formulated for standard linear applications, similar to the corresponding poverty measure.

$$hw_i = \{0,1\} \quad \text{identification: } hw_i = 1 \text{ for } x_i > w$$

$$iw_i = iw(w)_i = \left(\frac{x_i - \mathrm{w}}{w_{max} - \mathrm{w}}\right)^{\alpha} \text{ for } x_i \geq w$$

$$wFGT = \frac{1}{n} \sum_{r=1}^{n} hw_i \, iw_i$$

[7] Modified Aggregation of Poverty
(Non-Linear for [α=1])

$$hz_i = \{0,1\}$$

identification: $hz_i = 1$ for $y_i < z$

$$iz'_i = i(z)'_i = \left(\frac{\ln(iz1_i\ (z'-1)+1)}{(\ln z\prime)}\right)^{\alpha}$$

convex adaptation of poverty gaps for $0 < z' < 1$
concave adaptation of poverty gaps for $z' > 1$ *[default: z‘ = 100]*

$$zFGT' = \frac{1}{n}\sum_{r=1}^{n} hz_i\ iz'_i$$

[7] Modified Aggregation of Wealth
(Non-Linear for [α=1])

$$hw_i = \{0,1\}$$

identification: $hw_i = 1$ for $x_i > w$

$$iw'_i = i(w)'_i = \left(\frac{\ln(iw1_i\ (w'-1)+1)}{(\ln w\prime)}\right)^{\alpha}$$

convex adaptation of affluence for $0 < w' < 1$
concave adaptation of affluence for $w' > 1$ *[default: w‘ = 100]*

$$wFGT' = \frac{1}{n}\sum_{r=1}^{n} hw_i\ iw'_i$$

[8] Measuring (partial) Deprivation Levels

[Gaps for Partial Deprivation and Poverty]

The raw gap levels are calculated case as:

$$iz(z)_i = \left(\frac{z - y_i}{z - d_{min}}\right)^{\alpha} \text{ for } deprivation\ y_i \leq z$$

$$i(u)_i = \left(\frac{u - y_i}{u - d_{min}}\right)^{\alpha} \text{ for } partial\ deprivation\ y_i \leq u$$

[8] Measuring (partial) Affluence Levels

[Affluences (negative gaps) for Partial Wealth and Wealth]

The raw affluence levels are calculated as:

$$iw(w)_i = \left(\frac{x_i - \mathrm{w}}{w_{max} - \mathrm{w}}\right)^{\alpha} \quad \text{for wealth } x_i \geq w$$

$$i(l)_i = \left(\frac{x_i - \mathrm{l}}{w_{max} - \mathrm{l}}\right)^{\alpha} \quad \text{for partial wealth } x_i \geq l$$

[8a] Measuring (partial) Deprivation Levels

[Non-Linear Gaps for Partial Deprivation and Poverty (if α=1)]

… and for the non-linear case, regarding the baseline value for the upper limit (u') for partial deprivation, as:

$$iz(z)'_i = \left(\frac{\ln(i1(z)_i\,(z'-1)+1)}{(\ln z')}\right)^{\alpha}$$

for $i1(z)_i = i(z)_i$ for $\alpha = 1$

for concave (convex) adaptations $z' > (<)1$

$$i(u)'_i = \left(\frac{\ln(i1(u)_i\,(u'-1)+1)}{(\ln u')}\right)^{\alpha}$$

for $i1(u)_i = i(u)_i$ for $\alpha = 1$

for concave (convex) adaptations $u' > (<)1$

[8a] Measuring (partial) Affluence Levels

[Non-Linear Affluences (negative gaps) for Partial Wealth and Wealth (if α=1)]

… and for the non-linear case, regarding the baseline value for the lower limit (l') for partial wealth, as:

$$iw(w)'_i = \left(\frac{\ln(i1(w)_i\,(w'-1)+1)}{(\ln w')}\right)^{\alpha}$$

for $i1(w)_i = i(w)_i$ for $\alpha = 1$

for concave (convex) adaptations $w' > (<)1$

$$i(l)'_i = \left(\frac{\ln(i1(l)_i\,(l'-1)+1)}{(\ln l')}\right)^{\alpha}$$

for $i1(l)_i = i(l)_i$ for $\alpha = 1$

for concave (convex) adaptations $l' > (<)1$

[9] Adjustment of Deprivation Levels for Partial Deprivation

$$b^{-a*\alpha}$$

b ratio of poverty threshold to upper limit for partial deprivation
a sensitivity parameter {0,...,n} [default: a = 1]
α FGT parameter [0 = headcount]

The adjustment (down-weighting) of deprivation (gaps) in partial wealth can be solved for the linear case as

$$iz(u)_i = b^{-a*\alpha}\; i(u)_i$$

for partial deprivation $y_i \leq u$ [linear case (if a=1)]

… and in the same way also for non-linear applications.

$$iz(l)'_i = b^{-a*\alpha}\; i(u)'_i$$

for partial deprivation $y_i \leq u$ [non-linear case (if a=1)]

[9] Adjustment of Affluence Levels for Partial Wealth

$$q^{a*\alpha}$$

q ratio of wealth threshold to lower limit for partial wealth
a sensitivity parameter {0,...,n} [default: a = 1]
α FGT parameter [0 = headcount]

The adjustment (down-weighting) of affluence (neg.gaps) in partial wealth can be solved for the linear case as

$$iw(l)_i = q^{a*\alpha}\; i(l)_i$$

for partial wealth $x_i \geq l$ [linear case (if a=1)]

… and in the same way also for non-linear applications.

$$iw(l)'_i = q^{a*\alpha}\; i(l)'_i$$

for partial wealth $x_i \geq l$ [non-linear case (if a=1)]

[10] Inclusive Aggregation of Poverty and Partial Deprivation

$hz_i = 1$ if $y_i < z$
$hz_i = pd_i$ if $z \leq y_i < u$
$hz_i = 0$ if $y_i \geq u$

$iz_i = iz(z)_i$ if $y_i < z$ and $iz(z)_i \geq iz(u)_i$
$iz_i = iz(u)_i$ if $y_i < z$ and $iz(z)_i < iz(u)_i$
$iz_i = iz(u)_i$ if $z \leq y_i < u$
$iz_i = 0$ if $y_i \geq u$

with

y_i individual indicator for poverty
z poverty threshold (lower limit for fuzzy zone of precarity)
u precarity threshold (upper limit for fuzzy zone of precarity)
hz_i individual poverty score [0,1]
pd_i individual partial deprivation score within the fuzzy zone of precarity [0,1].

[10] Inclusive Aggregation of Wealth and Partial Wealth

$hw_i = 1$ if $x_i \geq w$
$hw_i = pw_i$ if $w > x_i \geq l$
$hw_i = 0$ if $x_i \leq l$

$iw_i = iw(w)_i$ if $x_i \geq w$ and $iw(w)_i \geq iw(l)_i$
$iw_i = iw(l)_i$ if $x_i \geq w$ and $iw(w)_i < iw(l)_i$
$iw_i = iw(l)_i$ if $w > x_i \geq l$
$iw_i = 0$ if $x_i \leq l$

with

x_i individual indicator for wealth
w wealth threshold (upper limit for fuzzy zone of prosperity)
l prosperity threshold (lower limit for fuzzy zone of prosperity)
hw_i individual wealth score [0,1]
pw_i individual partial wealth score within the fuzzy zone of prosperity [0,1].

[11] Aggregation of Poverty and Partial Deprivation

$$zFGT = \frac{1}{n} \sum_{i=1}^{n} hz_i \, iz_i$$

Intensity levels of gaps(iz_i) can each be replaced by nonlinear components (iz'_i) for both partial and non-partial elements, ($iz(z)'_i$; $(iu(u)'_i$).

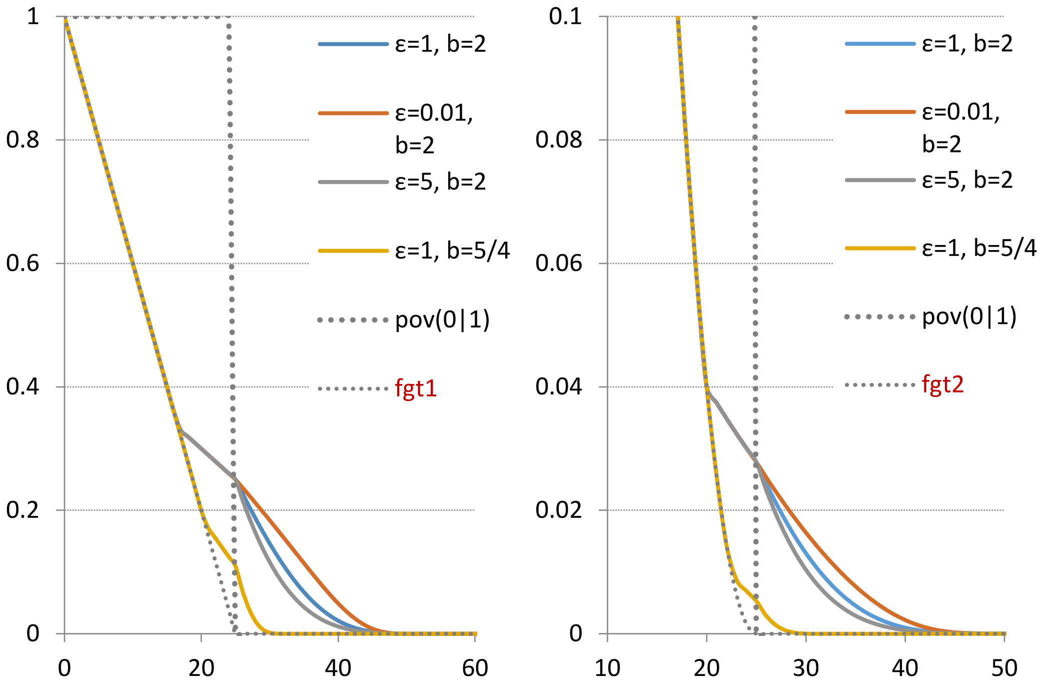
[11] Aggregation of Wealth and Partial Wealth

$$wFGT = \frac{1}{n} \sum_{i=1}^{n} hw_i \, iw_i$$

.

Intensity levels of affluence (iw_i) can each be replaced by nonlinear components (iw'_i) for both partial and non-partial elements, ($iw(w)'_i$; $(iw(l)'_i$).

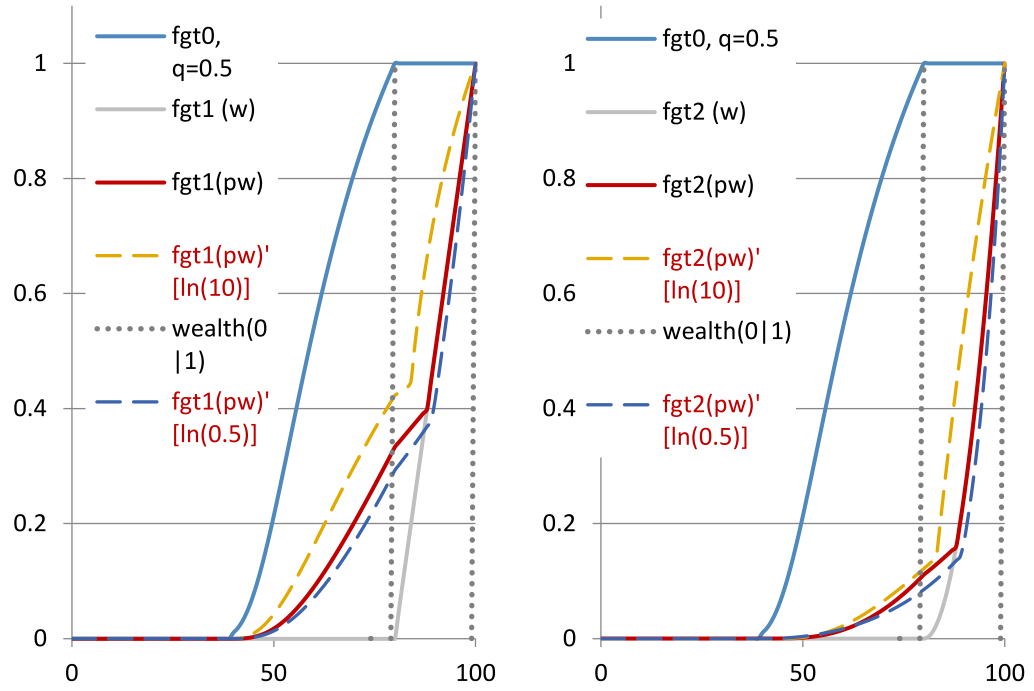
[12] Joint Cardinal FGT Measures for Intensity (gaps) of Poverty and Partial Deprivation*



*here: $i(u)_i = \left(\frac{1}{1+\alpha}\right)^{\alpha} \left(\frac{u-y_i}{u}\right)^{\alpha}$ $for\ y_i < u$

Instead of: $i(u)_i = b^{-a*\alpha} \left(\frac{u-y_i}{u-dmin}\right)^{\alpha}$ $for\ y_i < u$

[12] Joint Cardinal FGT Measures for Intensity (negative gaps) of Wealth and Partial Wealth



$i(l)_i = q^{a*\alpha} \left(\frac{x_i-l}{d_{max}-l}\right)^{\alpha}$ $for\ x_i > l$