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Description

lssolve (A, B) finds the minimum-norm least-squares solution for min $||A X - B||_2$ and returns X. A can be real or complex and can also be rank deficient. A does not have to be a square matrix.

lssolve(*A*, *B*, *rank*) does the same thing but also returns the effective rank in *rank*.

lssolve(A, B, rank, favorspeed) does the same thing but allows you to specify the computation method; see Computation methods and tolerance under Remarks and examples below.

lssolve (A, B, rank, favorspeed, tol) does the same thing but allows you to specify the tolerance for declaring the effective rank of A; see Computation methods and tolerance under Remarks and examples below.

 $_lssolve(A, B), _lssolve(A, B, favorspeed), and _lssolve(A, B, favorspeed, tol) do the same thing except that rather than returning the solution X, they overwrite B with the solution and return the effective rank. In the process of performing the calculation, they destroy the contents of A.$

_leastsquare_lapacke(A, B, rank), _leastsquare_lapacke(A, B, rank, favorspeed), and _leastsquare_lapacke(A, B, rank, favorspeed, tol) are the interfaces to the LAPACK routines that do the work. They find the minimum-norm solution for the least-squares problem $||AX - B||_2$, returning the solution in B and, in the process, using as workspace (overwriting) A. The routines return 0 if a solution was found and 1 otherwise. If 1 is returned, B is overwritten with a matrix of missing values.

Note that these functions can be used only when set lapack_mkl on is in effect on Windows or Linux or when set lapack_openblas on is in effect on Mac; see [M-1] LAPACK.

Syntax

numeric matrix	lssolve(A, B)
numeric matrix	<pre>lssolve(A, B, rank)</pre>
numeric matrix	<pre>lssolve(A, B, rank, favorspeed)</pre>
numeric matrix	<pre>lssolve(A, B, rank, favorspeed, tol)</pre>
real scalar	_lssolve(A, B)
real scalar	<pre>_lssolve(A, B, favorspeed)</pre>
real scalar	<pre>_lssolve(A, B, favorspeed, tol)</pre>
real scalar	<pre>_leastsquare_lapacke(A, B, rank)</pre>
real scalar	<pre>_leastsquare_lapacke(A, B, rank, favorspeed)</pre>
real scalar	_leastsquare_lapacke(A, B, rank, favorspeed, tol)

where inputs are

A:	numeric matrix
<i>B</i> :	numeric matrix
favorspeed:	real scalar
tol:	real scalar

and outputs are

B:	<i>numeric matrix</i> (solution of $A X = B$ overwritten in B)
rank:	real scalar
result:	real scalar

Remarks and examples

Remarks are presented under the following headings:

Introduction Computation methods and tolerance Examples

Introduction

The above functions solve A X = B via the least-squares method. A does not have to be square and can be rank deficient.

The least-squares method tries to find the minimum-norm solution of the following problem:

 $\min \|A X - B\|_2$

When A is square and of full rank, the computed solution is the same as

 $X = A^{-1}B$

When A is not square and not rank deficient, the computed solution is

$$X = (A'A)^{-1}A'B$$

when the number of rows of A is greater than the number of columns of A, and

$$X = A'(AA')^{-1}B$$

when the number of rows of A is less than the number of columns of A.

When A is rank deficient, the inverses in the above formulas are replaced by the generalized Moore–Penrose pseudoinverse. See [M-5] **pinv()** for more details about the pseudoinverse.

Computation methods and tolerance

When *favorspeed* is missing or 0, singular value decomposition is used. This is also the default method when *favorspeed* is not specified. It works with the optional argument *tol* to decide whether the matrix *A* is rank deficient.

When *favorspeed* is not missing and not 0, the QR or LQ factorization method is used. This method is faster but assumes A has full rank, so the optional argument *tol* is irrelevant in this case.

The default tolerance used is

$$\eta = \frac{(1e-13)*trace(abs(A))}{l}$$

where A is $m \times n$ and l is the minimum of m and n. A singular value of A is considered 0 if it is less than or equal to $tol \times the$ largest singular value of A.

If you specify tol > 0, the value you specify is used to multiply η . You may instead specify $tol \le 0$, and then the negative of the value you specify is used in place of η ; see [M-1] Tolerance.

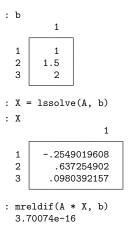
See [M-5] **lusolve()** for a detailed discussion of the issues surrounding solving nearly singular systems. The main point is that if A is ill conditioned, then small changes in A or B can lead to radically large differences in the solution for X.

Examples

Example 1: Square matrix

If A is square and has full rank, the minimum-norm least-squares solution computed by lssolve() is the same as $X = A^{-1}B$.

```
: A = (3, 2, 5 \setminus 2, 3, 1 \setminus 1, 2, 10)
: b = (1 \setminus 1.5 \setminus 2)
: A
           1
                  2
                          3
           3
                  2
                          5
  1
  2
           2
                  3
                        1
  3
           1
                  2
                        10
```



We can also check the effective rank by typing

```
: X = lssolve(A, b, rank = .)
: rank
3
```

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Example 2: Nonsquare matrix

When A is not square and is not rank deficient, the solution of the least-squares problem provided by lssolve(),

$$\min \|A X - B\|_2$$

is the same as $X = (A'A)^{-1}A'B$. We can confirm that we get the same result with both methods with the example below.

```
: A = (3, 2 \setminus 2, 1 \setminus 1, 20)
: b = (1 \setminus 1.5 \setminus 2)
: A
           1
                  2
  1
           3
                  2
  2
           2
                  1
  3
           1
                 20
: b
            1
  1
            1
  2
          1.5
  3
            2
: X = lssolve(A, b)
```

Now the effective rank is

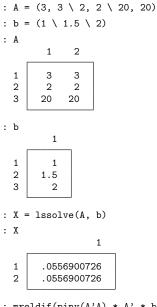
```
: X = lssolve(A, b, rank = .)
: rank
2
```

Example 3: Rank-deficient matrix

When A is not square but is rank deficient, we can find the solution to the least-squares problem

 $\min \|A X - B\|_2$

with lssolve() as well. Note that we cannot specify the parameter favorspeed this time.



: mreldif(pinv(A'A) * A' * b, X) 1.97186e-17 4

Now the effective rank is

: X = lssolve(A, b, rank = .) : rank 1

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Conformability

<pre>lssolve(A, B, rank, favour input:</pre>	rspeed, tol?):			
<i>A</i> :	$m \times n$				
<i>B</i> :	$m \times k$				
favorspeed:	1×1	(optional)			
tol:	1×1	(optional)			
output:					
rank:	1×1	(optional)			
result:	$n \times k$				
<pre>_lssolve(A, B, favorspeed, tol):</pre>					
input:					
A:	$m \times n$				
<i>B</i> :	$m \times k$				
favorspeed:		(optional)			
tol:	1×1	(optional)			
output:					
<i>A</i> :	0 imes 0				
<i>B</i> :	$n \times k$				
rank:	1×1				
_leastsquare_lapacke(A	l, B, rank,	favorspeed, tol) :			
input:					
<i>A</i> :	$m \times n$				
<i>B</i> :	$m \times k$				
favorspeed:	1×1	(optional)			
tol:	1×1	(optional)			
output:					
<i>A</i> :	0 imes 0				
<i>B</i> :	$n \times k$				
rank:	1×1	(optional)			
result:	1×1				

Diagnostics

 $lssolve(A, B, ...), _lssolve(A, B, ...), and _leastsquare_lapacke(A, B, ...) return a result containing missing if A or B contains missing values. The functions with a nonzero$ *favorspeed* $return a result containing all missing values if A is singular. The functions abort with error if set lapack_mkl on is not in effect on Windows or Linux or when set lapack_openblas on is not in effect on Mac.$

_lssolve(A, B, ...) and _leastsquare_lapacke(A, B, ...) abort with error if A or B is a view.

_leastsquare_lapacke(A, B, ...) should not be used directly; use _lssolve().

Also see

- [M-5] qrsolve() Solve AX=B for X using QR decomposition
- [M-5] svsolve() Solve AX=B for X using singular value decomposition
- [M-4] Matrix Matrix functions
- [M-4] Solvers Functions to solve AX=B and to obtain A inverse

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