

# Bayesian analysis

Your Bayesian analysis in Stata can be as simple or as complex as your research problem.

- Thousands of built-in models
- Add your own models
- Prefix your command with **bayes**:
- Adaptive Metropolis–Hastings
- Gibbs sampling
- Multiple chains
- Convergence diagnostics
- Explore distributions
- Model goodness of fit
- Posterior predictive  $p$ -values
- Posterior summaries
- Hypothesis testing
- Model comparison
- Predictions
- Model averaging **New**
- More

The screenshot shows the Stata Bayesian analysis interface. At the top, there's a command line with ". bayes, nchains(4): logistic z x1 x2 x3". Below it is a "Model summary" section with details like "Prior: {z:x1 x2 x3 \_cons} ~ normal(0,10000)". The main area displays "Simulating chains: 1 2 3 4 (done)" and a table of MCMC statistics. The table includes columns for "Number of chains", "Per MCMC chain", "Iterations", "Burnin", "Sample size", "Number of obs", "Avg acceptance rate", "Avg efficiency", and "Max Gelman-Rubin R<sub>c</sub>". Below the table, there are notes about baseline odds, default priors, and initial values. At the bottom right, there are buttons for "CAP", "NUM", and "INS".

## Fit regression models

Linear regression

```
. bayes: regress y x1 x2 x3
```

Logistic regression

```
. bayes: logistic z x1 x2 x3
```

Multilevel regression

```
. bayes: mixed y x1 x2 x3 || id:
```

Vector autoregression (VAR)

```
. bayes: var y1 y2 y3, lags(1/3) exog(x1 x2)
```

Specify multiple chains

```
. bayes, nchains(4): logistic z x1 x2 x3
```

## Fit general models

Multilevel meta-analysis model

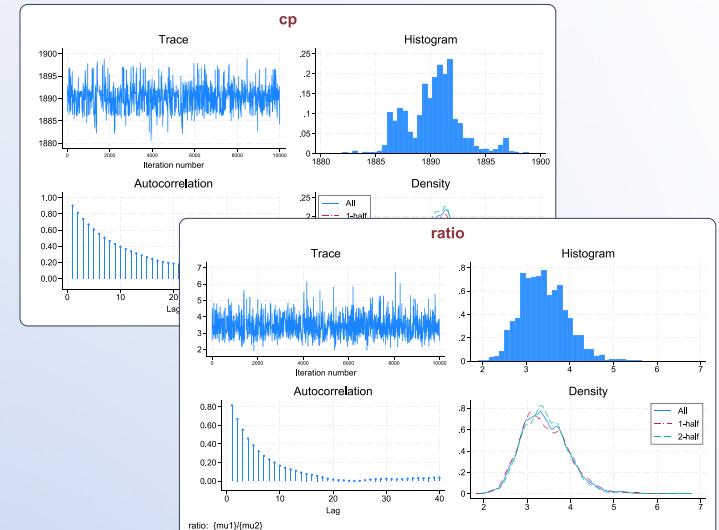
```
. bayesmh lnOR U[ttrial], noconstant likelihood(normal(var))
    prior({U[ttrial]}, normal({theta},{tau2}))
    prior({theta}, normal(0,10000))
    prior({tau2}, igamma(0.0001,0.0001))
    block({theta tau2}, gibbs split)
```

Nonlinear Poisson model: Change-point analysis

```
. bayesmh count, likelihood(dpoisson({mu1}*sign(year<{cp})+{mu2}*sign(year>={cp})))
    prior({mu1 mu2}, flat)
    prior({cp}, uniform(1851,1962))
    initial({mu1 mu2} 1 {cp} 1906)
```

## Check convergence

```
. bayesgraph diagnostics {cp}
    (ratio: {mu1}/{mu2})
```



## Program your own models

### Hurdle model

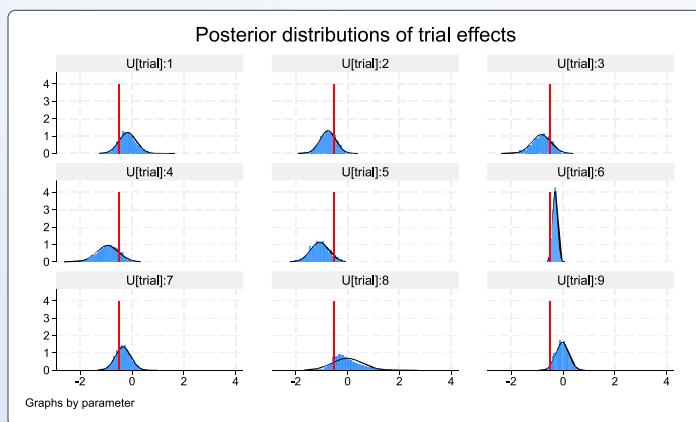
```
. bayesmh (hours age) (hours0 commute), llevaluator(mychurdle, parameters({lnsig}))  
prior({hours:} {hours0:} {lnsig}, flat)
```

```
program mychurdle  
version 18.0  
args lnf xb xg lnsig  
tempname sig  
scalar `sig' = exp(`lnsig')  
tempvar lnfj  
qui gen double `lnfj' = normal(`xg')  
qui replace `lnfj' = log(1 - `lnfj') if $MH_y1 <= 0  
qui replace `lnfj' = log(`lnfj') - log(normal(`xb'/'`sig')) +  
log(normalden($MH_y1,'`xb','`sig')) if $MH_y1 > 0  
summarize `lnfj', meanonly  
if r(N) < $MH_n {  
    scalar `lnf' = .  
    exit  
}  
scalar `lnf' = r(sum)  
end
```

## Perform inference

### Explore distributions

```
. bayesgraph histogram {U[ttrial]}, ...
```



### Test hypothesis

```
. bayestest interval {mu1}/{mu2}, lower(3)
```

Interval tests                            MCMC sample size = 10,000

prob1 : {mu1}/{mu2} > 3

	Mean	Std. dev.	MCSE
prob1	.7147	0.45158	.0216545

### Compare models

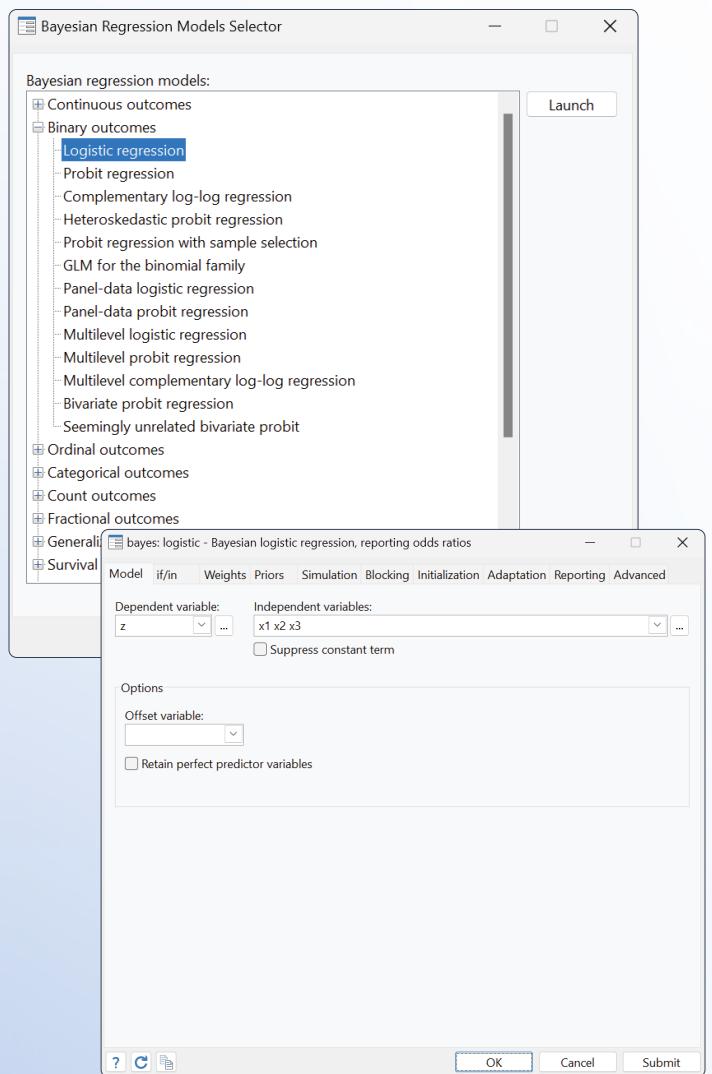
```
. bayesstats ic model1 model2
```

Bayesian information criteria

	DIC	log(ML)	log(BF)
model1	472.0359	-242.5827	.
model2	470.8157	-235.7438	6.838942

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

## Perform any analyses using GUI



# Regression models

Simply prefix your regression command with **bayes**:

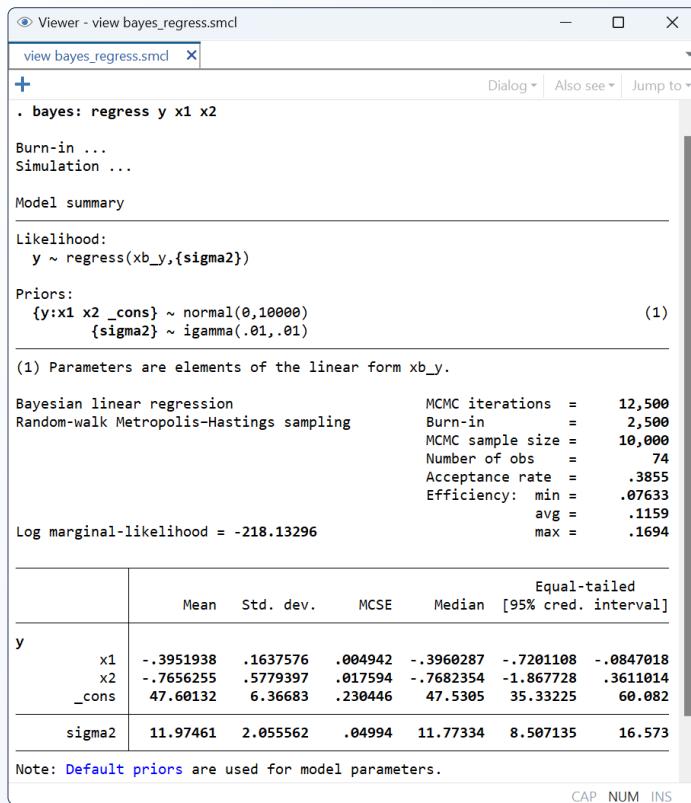
Continuous  
Binary  
Categorical  
**Multilevel models**  
Ordinal  
Censoring  
Truncation  
Sample selection  
Panel data  
Count  
Zero-inflated  
Survival

- Over 60 likelihood models supported, including multilevel, survival, GLM, VAR, DSGE, and more
- Censoring, truncation, sample selection
- Intuitive and elegant model specification
- Default and custom priors
- Comprehensive Bayesian-features support

## Linear regression

Use default normal priors for coefficients and inverse-gamma prior for variance

```
. bayes: regress y x1 x2
```



The screenshot shows the Stata 'Viewer' window with the command `. bayes: regress y x1 x2` entered. The output displays a 'Model summary' section with Likelihood and Priors information. It also shows Bayesian linear regression results with MCMC iterations, Burn-in, and acceptance rate. A table provides parameter estimates for y, x1, x2, and \_cons. A note at the bottom states: 'Note: Default priors are used for model parameters.'

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]
y					
x1	-.3951938	.1637576	.004942	-.3960287	-.7201108 -.0847018
x2	-.7656255	.5779397	.017594	-.7682354	-1.867728 .3611014
_cons	47.60132	6.36683	.230446	47.5305	35.33225 60.082
sigma2	11.97461	2.055562	.04994	11.77334	8.507135 16.573

Use Gibbs sampling

```
. bayes, gibbs: regress y x1 x2
```

## Logistic regression

Use default normal priors for coefficients

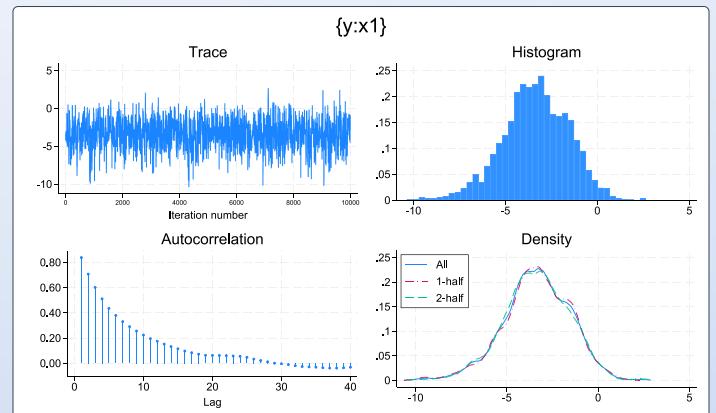
```
. bayes: logistic y x1 x2
```

Use custom Cauchy priors for coefficients on **x1** and **x2**

```
. bayes, prior({y:x1 x2}, cauchy(0,2.5)): logistic y x1 x2
```

Check convergence of coefficient on **x1**

```
. bayesgraph diagnostics {y:x1}
```



## Generalized linear model

Use burn-in of 1,000 and MCMC size of 5,000

```
. bayes, burnin(1000) mcmcsize(5000):  
    glm y x1 x2, family(binomial) link(log)
```

Test that coefficient `{y:x1}` is greater than 4

```
. bayestest interval {y:x1}, lower(4)
```

	Mean	Std. dev.	MCSE
prob1	.7881	.40867	.0125451

## Survival regression

Declare survival data

```
. stset time, failure(died)
```

Fit Bayesian exponential regression

```
. bayes, saving(mcmc_exp): streg x1 x2,  
    distribution(exponential)
```

```
. estimates store exp
```

Fit Bayesian Weibull regression

```
. bayes, saving(mcmc_weibull): streg x1 x2,  
    distribution(weibull)
```

```
. estimates store weibull
```

Compare models using the Bayes factor

```
. bayesstats ic exp weibull, bayesfactor
```

	DIC	log(ML)	BF
exp	103.4405	-71.04365	.
weibull	92.02015	-74.79336	.0235246

Compare models using posterior probabilities

```
. bayestest model exp weibull
```

	log(ML)	P(M)	P(M y)
exp	-71.0436	0.5000	0.9770
weibull	-74.7934	0.5000	0.0230

## Other regression models

Ordered logistic regression

```
. bayes: ologit y x1 x2
```

Conditional logistic regression

```
. bayes: clogit y x1 x2, group(id)
```

Poisson regression

```
. bayes: poisson y x1 x2
```

Truncated Poisson regression

```
. bayes: tpoisson y x1 x2, 11(10)
```

Zero-inflated negative binomial regression

```
. bayes: zinb y x1 x2, inflated(z1 z2)
```

Tobit regression

```
. bayes: tobit y x1 x2, ul(20)
```

Heteroskedastic probit regression

```
. bayes: hetprobit y x1 x2, het(xhet)
```

Heckman selection model

```
. bayes: heckman y x1 x2, select(x1 x2 x3)
```

Multivariate regression

```
. bayes: mvreg y1 y2 y3 = x1 x2
```

Multilevel regression

```
. bayes: mixed y x1 x2 || id:
```

Vector autoregression (VAR)

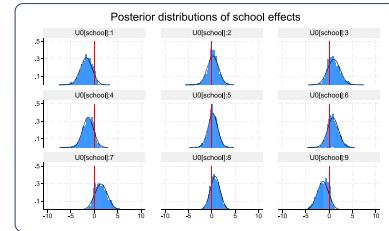
```
. bayes: var y1 y2 y3, lags(1/3) exog(x1 x2)
```

And more

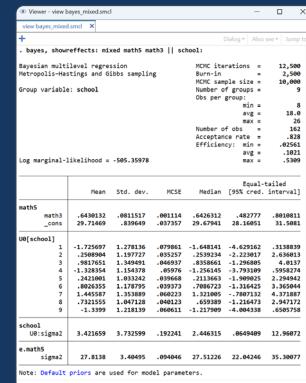
```
. bayes: ...
```

# Multilevel models

- Small number of groups?
- Many hierarchical levels?
- Want posterior distributions of random effects?



- Continuous, censored, binary, ordinal, and count outcomes
- Support for GLM and survival methods
- Random intercepts and coefficients
- Nested and crossed effects
- Multiple levels of hierarchy
- Random-effects covariance structures
- Multivariate nonlinear multilevel models
- Comprehensive Bayesian-features support



## Two-level models: Random intercepts

Fit regression of **math5** on **math3** with random intercepts by **school**

```
. bayes: mixed math5 math3 || school:
```

Display estimates of random effects

```
. bayes, showreffects:  
mixed math5 math3 || school:
```

(See output above)

Specify custom uniform priors instead of default normal priors for coefficients

```
. bayes, prior({math5:math3 _cons},  
uniform(-50,50)):  
mixed math5 math3 || school:
```

Plot posterior distributions of random intercepts

```
. bayesgraph histogram {U0}, byparm
```

(See graph above)

## Two-level models: Random coefficients

Add random coefficient on **math3** by **school**

```
. bayes: mixed math5 math3 || school: math3
```

Specify unstructured covariance for random effects

```
. bayes: mixed math5 math3 || school: math3,  
covariance(unstructured)
```

## Three-level models

Add random intercepts for teachers nested within schools

```
. bayes: mixed math5 math3 || school: || teacher:
```

## Crossed-effects models

Include crossed random effects of primary and secondary schools

```
. bayes: mixed math5 math3 ||  
_all: R.primary || secondary:
```

## Other multilevel models

Logistic regression

```
. bayes: meologit y x1 x2 || id:
```

Poisson regression

```
. bayes: mepoisson y x1 x2 || id:
```

Generalized linear model

```
. bayes: meglm y x1 x2 || id:,  
family(binomial) link(cloglog)
```

Ordered logistic regression

```
. bayes: meologit y x1 x2 || id:
```

Survival regression

```
. bayes: mestreg x1 x2 || id:,  
distribution(weibull)
```

And more

```
. bayes: any multilevel command ...
```

# Multiple chains, predictions, and more

- Multiple chains
- Gelman–Rubin convergence diagnostics
- Bayesian predictions

- Posterior summaries of simulated values
- MCMC replicates
- Posterior predictive  $p$ -values

## Two-level models: Random coefficients

Use option **nchains()** with **bayes:** or **bayesmh** to simulate multiple chains

Fit regression of **y** on covariates **x1** through **x10** and generate 3 chains

```
Viewer - view bayes_chains.smcl
view bayes_chains.smcl

+ . bayes, nchains(3): regress y x1-x10

Chain 1
Burn-in ...
Simulation ...

Chain 2
Burn-in ...
Simulation ...

Chain 3
Burn-in ...
Simulation ...

Model summary

Likelihood:
y ~ regress(xb_y,{sigma2})

Priors:
{y:x1 x2 x3 x4 x5 x6 x7 x8 x9 x10 _cons} ~ normal(0,1e000) (1)
{sigma2} ~ igamma(.01,.01)

(1) Parameters are elements of the linear form xb_y.

Bayesian linear regression
Random-walk Metropolis-Hastings sampling
Number of chains = 3
Per MCMC chain:
Iterations = 12,500
Burn-in = 2,500
Sample size = 10,000
Number of obs = 442
Avg acceptance rate = .321
Avg efficiency: min = .003771
avg = .01886
max = .1142
Avg log marginal-likelihood = -2457.6885
Max Gelman-Rubin Rc = 4.5435


```

	Mean	Std. dev.	MCSE	Median	Equal-tailed [95% cred. interval]
<b>y</b>					
x1	81.50951	87.57016	4.06945	71.7336	-50.84804 234.4335
x2	-167.0296	71.9136	6.76093	-167.887	-289.098 -42.73281
x3	352.3836	99.94654	6.13795	360.123	193.378 505.5432
x4	286.0075	78.16619	5.23075	275.4511	174.2242 426.8137
x5	-273.1433	164.7453	14.618	-295.8386	-501.8144 -16.60783
x6	165.232	178.513	8.84052	231.598	-125.8265 352.7025
x7	-94.36373	114.5865	7.16308	-106.0647	-263.5826 99.65676
x8	109.925	162.0595	13.0143	134.8212	-155.8764 332.5971
x9	483.243	102.1253	6.00414	495.4546	307.5461 627.0424
x10	42.76467	117.208	6.29098	65.24344	-146.8923 198.615
_cons	152.4957	2.780968	.103842	152.4606	147.3315 157.9954
<b>sigma2</b>	3110.56	221.9696	3.79219	3100.067	2707.435 3566.951

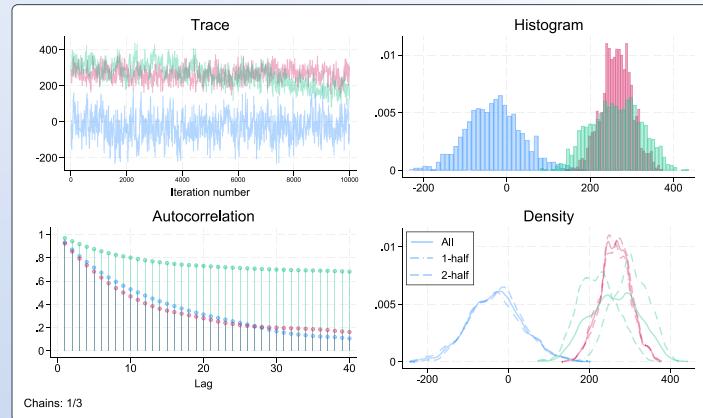
Note: Default priors are used for model parameters.  
Note: Default initial values are used for multiple chains.  
Note: There is a high autocorrelation after 500 lags in at least one of the chains.

## Gelman–Rubin convergence diagnostics

Check Gelman–Rubin convergence diagnostics

Explore convergence visually for coefficient of **x6**

. **bayesgraph diagnostics {y:x6}**



# Bayesian predictions

- Predict new values
- Check model fit using posterior predictive checks
- Compute functions of predicted values
- Specify your own prediction functions

- Obtain posterior summaries of predicted values
- Generate MCMC replicates
- Compute posterior predictive *p*-values

Bayesian predictions are outcome values simulated from the posterior predictive distribution. They are useful for predicting new outcome values and for checking model fit. Let's use **bayesmh** to fit a general Bayesian model.

```
. bayesmh y ..., likelihood(...) prior(...)
```

## Posterior summaries of predictions

Compute posterior mean and credible intervals for all observations, and store them in variables **pmean**, **cril**, and **criu**

```
. bayespredict pmean, mean  
. bayespredict criu criu, cri
```

	y	pmean	criu	cril
1.	2.933	3.111	4.230	2.014
2.	4.614	4.478	5.576	3.362
3.	1.654	2.034	3.115	0.936
4.	2.025	2.234	3.362	1.130
5.	3.165	2.894	4.014	1.790
6.	1.372	2.337	3.452	1.227
7.	2.921	3.253	4.372	2.127
8.	2.699	2.274	3.359	1.158
9.	1.198	1.228	2.312	0.124
10.	3.097	2.767	3.872	1.655

## MCMC replicates

Compute 6 MCMC replicates, and store them in variables **yrep1**, **yrep2**, and so on

```
. bayesreps yrep*, nreps(6)
```

List the first 10 observations

	y	yrep1	yrep2	yrep3	yrep4	yrep5	yrep6
1.	2.933	3.496	1.416	3.852	2.667	3.621	3.229
2.	4.614	4.794	3.462	4.354	6.245	3.848	4.822
3.	1.654	2.068	2.136	1.949	1.395	2.894	2.613
4.	2.025	2.568	2.234	2.780	1.966	1.804	2.230
5.	3.165	2.980	2.180	3.610	2.075	2.526	1.754
6.	1.372	1.584	2.110	2.932	0.956	2.149	2.438
7.	2.921	4.087	3.161	3.570	2.687	4.051	3.766
8.	2.699	1.731	1.846	2.216	2.065	2.109	1.994
9.	1.198	1.615	1.039	1.530	0.612	1.092	1.478
10.	3.097	2.281	2.774	2.799	2.162	4.188	3.107

## Posterior predictive *p*-values

Simulate predictions for outcome **y**, and save them in **y\_pred.dta**

```
. bayespredict {_ysim}, saving(y_pred)
```

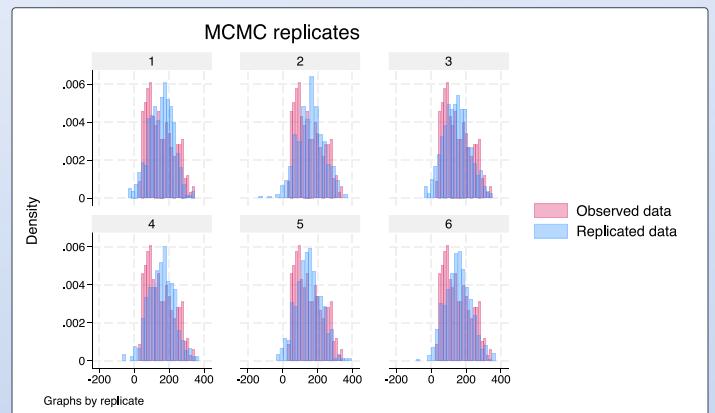
Compute posterior predictive *p*-values; use Mata's built-in functions and your own

T	Mean	Std. dev.	E(T_obs)	P(T>=T_obs)
mean	3.045143	.0787588	3.044554	.5026
min	5.130189	.3401942	1.040675	.0365
max	5.84896	.3703789	5.703145	.626
skew	.1471358	.16660461	.1555046	.4806

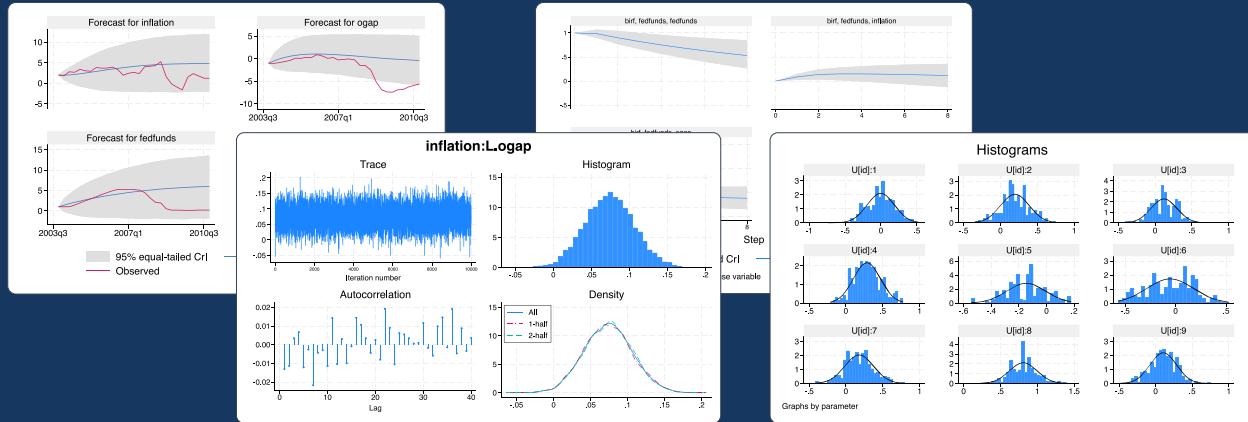
Note: P(T>=T\_obs) close to 0 or 1 indicates lack of fit.

```
mata:  
real scalar myskew(real colvector x)  
> return (sqrt(length(x))*sum((x:-mean(x)):^(3:))/sum((x:-mean(x)):^(2:)^1.5))  
>  
end
```

## Plot distributions of MCMC replicates



# Bayesian econometrics

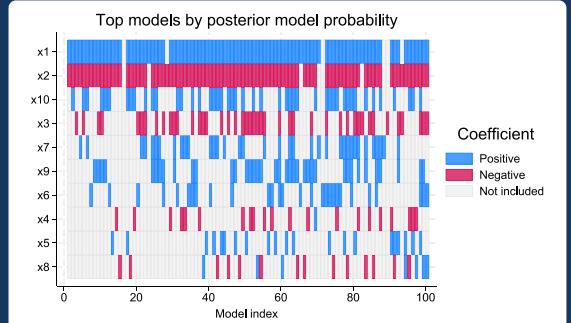


- Panel-data models
- VAR models
- Dynamic forecasting
- IRF and FEVD analysis
- Linear and nonlinear DSGE models
- And more

[stata.com/bayesian-econometrics](https://stata.com/bayesian-econometrics)

## New in Stata 18

- Bayesian model averaging (BMA)
  - BMA for linear regression
  - Influential models and important predictors
  - Posterior distribution plots for regression coefficients
  - Model-probability plots
  - Variable-inclusion maps
  - Model fit and predictive performance
- Bayesian quantile regression [StataNow](#)
- Bayesian asymmetric Laplace Model [StataNow](#)



[stata.com/new-in-bayesian-analysis](https://stata.com/new-in-bayesian-analysis)

[stata.com/bayesian-analysis](https://stata.com/bayesian-analysis)