## Just tired of endless loops! or parallel: Stata module for parallel computing

George G. Vega Yon<sup>1</sup> Brian Quistorff<sup>2</sup>

<sup>1</sup>University of Southern California vegayon@usc.edu

<sup>2</sup>Microsoft AI and Research Brian.Quistorff@microsoft.com

Stata Conference Baltimore July 27–28, 2017

Thanks to Stata users worldwide for their valuable contributions. The usual disclaimers applies.

## Agenda

Motivation

What is it and how does it work

Benchmarks

Syntax and Usage

**Concluding Remarks** 



Both computation power and size of data are ever increasing

- Both computation power and size of data are ever increasing
- > Often our work is easily broken down into independent chunks

▲□▶ ▲圖▶ ▲圖▶ ▲圖▶ = ● ● ●

- Both computation power and size of data are ever increasing
- > Often our work is easily broken down into independent chunks
- Implementing parallel computing, even for these "embarrassingly parallel" problems, however, is not easy.

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

- Both computation power and size of data are ever increasing
- > Often our work is easily broken down into independent chunks
- Implementing parallel computing, even for these "embarrassingly parallel" problems, however, is not easy.
- Stata/MP exists, but only parallelizes a limited set of internal commands, not user commands.

- Both computation power and size of data are ever increasing
- Often our work is easily broken down into independent chunks
- Implementing parallel computing, even for these "embarrassingly parallel" problems, however, is not easy.
- Stata/MP exists, but only parallelizes a limited set of internal commands, not user commands.

parallel aims to make this more convenient.

What is it and how does it work

#### Benchmarks

Syntax and Usage

**Concluding Remarks** 



 Inspired by the R package "snow" (several other examples exists: HTCondor, Matlab's Parallel Toolbox, etc.)

- Inspired by the R package "snow" (several other examples exists: HTCondor, Matlab's Parallel Toolbox, etc.)
- Launches "child" batch-mode Stata processes across multiple processors (e.g. simultaneous multi-threading, multiple cores, sockets, cluster nodes).

- Inspired by the R package "snow" (several other examples exists: HTCondor, Matlab's Parallel Toolbox, etc.)
- Launches "child" batch-mode Stata processes across multiple processors (e.g. simultaneous multi-threading, multiple cores, sockets, cluster nodes).
- Depending on the task, can reach near linear speedups proportional to the number of processors.

- Inspired by the R package "snow" (several other examples exists: HTCondor, Matlab's Parallel Toolbox, etc.)
- Launches "child" batch-mode Stata processes across multiple processors (e.g. simultaneous multi-threading, multiple cores, sockets, cluster nodes).
- Depending on the task, can reach near linear speedups proportional to the number of processors.

▶ Thus having a quad-core computer can lead to a 400% speedup.

## Simple usage

Serial:

- ▶ gen v2 = v\*v
- do byobs\_calc.do
- bs, reps(5000): reg price foreign rep

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

## Simple usage

Serial:

- ▶ gen v2 = v\*v
- do byobs\_calc.do
- bs, reps(5000): reg price foreign rep

Parallel:

- parallel: gen v2 = v\*v
- parallel do byobs\_calc.do
- parallel bs, reps(5000): reg price foreign rep

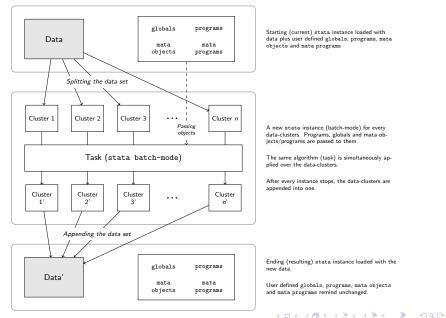
## What is it and how does it work

How does it work?

• Method is *split-apply-combine* like MapReduce.

### What is it and how does it work

How does it work?



▶ Method is *split-apply-combine* like MapReduce. Very flexible!

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>

- Method is split-apply-combine like MapReduce. Very flexible!
- Straightforward usage when there is observation- or group-level work

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

- Method is split-apply-combine like MapReduce. Very flexible!
- Straightforward usage when there is observation- or group-level work
- If each iteration needs the entire dataset, then use procedure to split the tasks and load the data separately. Examples:

- Method is split-apply-combine like MapReduce. Very flexible!
- Straightforward usage when there is observation- or group-level work
- If each iteration needs the entire dataset, then use procedure to split the tasks and load the data separately. Examples:

▲□▶ ▲□▶ ▲□▶ ▲□▶ □ のQ@

Table of seeds for each bootstrap resampling

- Method is split-apply-combine like MapReduce. Very flexible!
- Straightforward usage when there is observation- or group-level work
- If each iteration needs the entire dataset, then use procedure to split the tasks and load the data separately. Examples:

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

- Table of seeds for each bootstrap resampling
- Table of parameter values for simulations

- Method is split-apply-combine like MapReduce. Very flexible!
- Straightforward usage when there is observation- or group-level work
- If each iteration needs the entire dataset, then use procedure to split the tasks and load the data separately. Examples:

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

- Table of seeds for each bootstrap resampling
- Table of parameter values for simulations
- If the list of tasks is data-dependent then the "nodata" alternative mechanism allows for more flexibility.

#### Implementation Some details

 Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:

#### Implementation Some details

 Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:

◆□ ▶ < 圖 ▶ < 圖 ▶ < 圖 ▶ < 圖 • 의 Q @</p>

Functionality when the parent Stata is in batch-mode

#### Implementation Some details

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)

► For a Linux/MacOS cluster with a shared filesystem (e.g. NFS) and ssh-like commands, can distribute across nodes.

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)
- ► For a Linux/MacOS cluster with a shared filesystem (e.g. NFS) and ssh-like commands, can distribute across nodes.
  - New feature so we'd appreciate help from the community to extend to other cluster settings (e.g. PBS)

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)
- ► For a Linux/MacOS cluster with a shared filesystem (e.g. NFS) and ssh-like commands, can distribute across nodes.
  - New feature so we'd appreciate help from the community to extend to other cluster settings (e.g. PBS)

Make sure that child tempnames or tempvars don't clash with those coming from parent.

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)
- ► For a Linux/MacOS cluster with a shared filesystem (e.g. NFS) and ssh-like commands, can distribute across nodes.
  - New feature so we'd appreciate help from the community to extend to other cluster settings (e.g. PBS)

- Make sure that child tempnames or tempvars don't clash with those coming from parent.
- Passes through programs, macros and mata objects, but NOT Stata matrices or scalars. No state but datasets are returned to parent.

- Uses shell on Linux/MacOS. On Windows we have a compiled plugging allowing:
  - Functionality when the parent Stata is in batch-mode
  - Seamless user experience by launching the child programs in a hidden desktop (otherwise GUI for each steals focus)
- ► For a Linux/MacOS cluster with a shared filesystem (e.g. NFS) and ssh-like commands, can distribute across nodes.
  - New feature so we'd appreciate help from the community to extend to other cluster settings (e.g. PBS)

- Make sure that child tempnames or tempvars don't clash with those coming from parent.
- Passes through programs, macros and mata objects, but NOT Stata matrices or scalars. No state but datasets are returned to parent.
- ▶ Recover gracefully from child failures. Currently no re-try support.

What is it and how does it work

#### Benchmarks

Syntax and Usage

**Concluding Remarks** 



Bootstrap with parallel bs

sysuse auto, clear expand 10

// Serial fashion
bs, rep(\$size) nodots: regress mpg weight gear foreign

// Parallel fashion
parallel setclusters \$number\_of\_clusters
parallel bs, rep(\$size) nodots: regress mpg weight gear foreign

Bootstrap with parallel bs

sysuse auto, clear expand 10

// Serial fashion
bs, rep(\$size) nodots: regress mpg weight gear foreign

// Parallel fashion
parallel setclusters \$number\_of\_clusters
parallel bs, rep(\$size) nodots: regress mpg weight gear foreign

Problem size	Serial	2 Clusters	4 Clusters
1,000	2.93s	1.62s	1.09s
	×2.69	×1.48	×1.00
2,000	5.80s	3.13s	2.03s
	×2.85	×1.54	×1.00
4,000	11.59s	6.27s	3.86s
	×3.01	×1.62	$\times 1.00$

Table: Absolute and relative computing times for each run of a basic bootstrap problem. For each given problem size, the first row shows the time in seconds, and the second row shows the relative time each method took to complete the task relative to using parallel with four clusters. Each cell represents a 1,000 runs.

Simulations with parallel sim

```
prog def mysim, rclass
    // Data generating process
    drop _all
    set obs 1000
    gen eps = rnormal()
    gen X = rnormal()
    gen Y = X*2 + eps
    // Estimation
    reg Y X
    mat def ans = e(b)
    return scalar beta = ans[1,1]
end
```

```
// Serial fashion
simulate beta=r(beta), reps($size) nodots: mysim
```

```
// Parallel fashion
parallel setclusters $number_of_clusters
parallel sim, reps($size) expr(beta=r(beta)) nodots: mysim
```

Simulations with parallel sim (cont.)

Problem size	Serial	2 Clusters	4 Clusters
1000	2.19s ×3.01	$1.18s \times 1.62$	0.73s imes 1.00
2000	4.36s ×3.29	2.29s ×1.73	$\begin{array}{c} 1.33 \text{s} \\ \times 1.00 \end{array}$
4000	8.69s ×3.40	4.53s ×1.77	2.55s ×1.00

Table: Absolute and relative computing times for each run of a simple Monte Carlo exercise. For each given problem size, the first row shows the time in seconds, and the second row shows the relative time each method took to complete the task relative to using parallel with four clusters. Each cell represents a 1,000 runs.

▲ロト ▲帰 ト ▲ ヨ ト ▲ ヨ ト ・ ヨ ・ の Q ()

Code for replicating this is available at https://github.com/gvegayon/parallel

What is it and how does it work

Benchmarks

Syntax and Usage

**Concluding Remarks** 



Setup

parallel setclusters #|default [, force hostnames(namelist)]

▲□▶ ▲圖▶ ▲圖▶ ▲圖▶ = ● ● ●

Setup

parallel setclusters #|default [, force hostnames(namelist)]

Main command types

Setup

parallel setclusters #|default [, force hostnames(namelist)]

Main command types

### Setup

parallel setclusters #|default [, force hostnames(namelist)]

#### Main command types

Helper commands

### Setup

parallel setclusters #|default [, force hostnames(namelist)]

#### Main command types

Helper commands

### Setup

parallel setclusters #|default [, force hostnames(namelist)]

#### Main command types

Helper commands

```
parallel bs [, expression(exp_list) programs(namelist) mata seeds(string)
randtype(random.org|datetime) bs_options]: stata_omd
```

### Setup

parallel setclusters #|default [, force hostnames(namelist)]

#### Main command types

Helper commands

### Additional Utilities

parallel version/clean/printlog/viewlog/numprocessors



Use parallel printlog/viewlog to view the log of the child process (includes some setup code as well). Can set trace in the child do-file or command to see more.

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

# Debugging

Use parallel printlog/viewlog to view the log of the child process (includes some setup code as well). Can set trace in the child do-file or command to see more.

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 の�?

Auxiliary files created during process (harder to use):

# Debugging

Use parallel printlog/viewlog to view the log of the child process (includes some setup code as well). Can set trace in the child do-file or command to see more.

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

- Auxiliary files created during process (harder to use):
  - (Unix) \_\_pllID\_shell.sh
  - \_\_pllID\_dataset.dta
  - \_\_pll/D\_doNUM.do
  - \_\_pll*ID*\_glob.do
  - \_\_pll/D\_dtaNUM.dta
  - \_\_pllID\_finitoNUM

# Debugging

- Use parallel printlog/viewlog to view the log of the child process (includes some setup code as well). Can set trace in the child do-file or command to see more.
- Auxiliary files created during process (harder to use):
  - (Unix) \_\_pllID\_shell.sh
  - \_\_pllID\_dataset.dta
  - \_\_pll/D\_doNUM.do
  - \_\_pll*ID*\_glob.do
  - \_\_pllID\_dtaNUM.dta
  - \_\_pll/D\_finitoNUM
- Can keep these around by specifying the keep or keeplast options

▲ロト ▲帰ト ▲ヨト ▲ヨト 三日 - の々ぐ

parallel suits ...

Repeated simulation

◆□▶ ◆□▶ ◆臣▶ ◆臣▶ 臣 のへぐ

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)

◆□▶ ◆□▶ ◆三▶ ◆三▶ 三三 のへぐ

Bootstrapping/Jackknife

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)
- Bootstrapping/Jackknife
- Multiple MCMC chains to test for convergence (Gelman-Rubin test)

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)
- Bootstrapping/Jackknife
- Multiple MCMC chains to test for convergence (Gelman-Rubin test)

parallel doesn't suit ...

(already) fast commands

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)
- Bootstrapping/Jackknife
- Multiple MCMC chains to test for convergence (Gelman-Rubin test)

parallel doesn't suit ...

- (already) fast commands
- Regressions, ARIMA, etc.

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)
- Bootstrapping/Jackknife
- Multiple MCMC chains to test for convergence (Gelman-Rubin test)

parallel doesn't suit ...

- (already) fast commands
- Regressions, ARIMA, etc.

Linear Algebra

parallel suits ...

- Repeated simulation
- Extensive nested control flow (loops, while, ifs, etc.)
- Bootstrapping/Jackknife
- Multiple MCMC chains to test for convergence (Gelman-Rubin test)

parallel doesn't suit ...

- (already) fast commands
- Regressions, ARIMA, etc.
- Linear Algebra
- Whatever Stata/MP does better (on single machine)

## Use in other Stata modules

- EVENTSTUDY2: Perform event studies with complex test statistics
- MIPARALLEL: Perform parallel estimation for multiple imputed datasets

 Synth\_Runner: Performs multiple Synthetic Control estimations for permutation testing Brings parallel computing to many more commands than Stata/MP

▲□▶ ▲圖▶ ▲圖▶ ▲圖▶ = ● ● ●

Brings parallel computing to many more commands than Stata/MP

Its major strengths/advantages are in simulation models and non-vectorized operations such as control-flow statements.

- Brings parallel computing to many more commands than Stata/MP
- Its major strengths/advantages are in simulation models and non-vectorized operations such as control-flow statements.
- Depending on the proportion of the algorithm that can be parallelized, it is possible to reach near to linear scale speedups.

- Brings parallel computing to many more commands than Stata/MP
- Its major strengths/advantages are in simulation models and non-vectorized operations such as control-flow statements.
- Depending on the proportion of the algorithm that can be parallelized, it is possible to reach near to linear scale speedups.

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <

We welcome other user commands optionally utilizing parallel for increased performance.

- Brings parallel computing to many more commands than Stata/MP
- Its major strengths/advantages are in simulation models and non-vectorized operations such as control-flow statements.
- Depending on the proportion of the algorithm that can be parallelized, it is possible to reach near to linear scale speedups.

- We welcome other user commands optionally utilizing parallel for increased performance.
- Install, contribute, find help, and report bugs at http://github.com/gvegayon/parallel

## Thank you very much!

## George G. Vega Yon<sup>1</sup> Brian Quistorff<sup>2</sup>

<sup>1</sup>University of Southern California vegayon@usc.edu

<sup>2</sup>Microsoft AI and Research Brian.Quistorff@microsoft.com

Stata Conference Baltimore July 27–28, 2017

< □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > < □ > <