

**irt hybrid** — Hybrid IRT models

|                             |                                      |                                |                                      |
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## Description

`irt hybrid` fits IRT models to combinations of binary, ordinal, and nominal items.

## Quick start

1PL model for binary items `b1` to `b5` and 2PL model for binary items `b6` to `b10`

```
irt hybrid (1pl b1-b5) (2pl b6-b10)
```

Plot ICCs for each item

```
irtgraph icc
```

GRM for ordinal items `o1`, `o2`, and `o3`, 3PL model for binary items `b1` and `b2`, and NRM for nominal items `n1` to `n5`

```
irt hybrid (grm o1 o2 o3) (3pl b1 b2) (nrm n1-n5)
```

Plot CCCs for `o1`

```
irtgraph icc o1
```

## Menu

Statistics > IRT (item response theory)

# Syntax

```
irt hybrid (model varlist1) (model varlist2) [...] [if] [in] [weight] [, options]
```

| <i>model</i>          | Description                      |
|-----------------------|----------------------------------|
| <a href="#">1pl</a>   | One-parameter logistic model     |
| <a href="#">2pl</a>   | Two-parameter logistic model     |
| * <a href="#">3pl</a> | Three-parameter logistic model   |
| <a href="#">grm</a>   | Graded response model            |
| <a href="#">nrm</a>   | Nominal response model           |
| <a href="#">pcm</a>   | Partial credit model             |
| <a href="#">gpcm</a>  | Generalized partial credit model |
| <a href="#">rsm</a>   | Rating scale model               |

\* The full syntax for 3pl is (3pl *varlist* [, [sepguessing](#)]). Option [sepguessing](#) is documented in [\[IRT\] irt 3pl](#).

*varlist*<sub>1</sub>, *varlist*<sub>2</sub>, ..., *varlist*<sub>#</sub> may not contain duplicate items.

| <i>options</i>                               | Description   |
|--|---|
| Model  |   |
| <a href="#">listwise</a>                     | drop observations with any missing items  |
| SE/Robust                                    |   |
| <a href="#">vce(<i>vcetype</i>)</a>          | <i>vcetype</i> may be <a href="#">oim</a> , <a href="#">robust</a> , <a href="#">cluster <i>clustvar</i></a> , <a href="#">bootstrap</a> , or <a href="#">jackknife</a> |
| Reporting                                    |   |
| <a href="#">level(#)</a>                     | set confidence level; default is <a href="#">level(95)</a>  |
| <a href="#">notable</a>                      | suppress coefficient table  |
| <a href="#">noheader</a>                     | suppress output header  |
| <a href="#">display_options</a>              | control columns and column formats  |
| Integration                                  |   |
| <a href="#">intmethod(<i>intmethod</i>)</a>  | integration method  |
| <a href="#">intpoints(#)</a>                 | set the number of integration points; default is <a href="#">intpoints(7)</a>   |
| Maximization                                 |   |
| <a href="#">maximize_options</a>             | control the maximization process; seldom used   |
| <a href="#">startvalues(<i>svmethod</i>)</a> | method for obtaining starting values  |
| <a href="#">noestimate</a>                   | do not fit the model; show starting values instead  |
| <a href="#">dnnumerical</a>                  | use numerical derivative techniques   |
| <a href="#">coeflegend</a>                   | display legend instead of statistics  |

| <i>intmethod</i>            | Description  |
|-----------------------------|--|
| <a href="#">mvaghermite</a> | mean–variance adaptive Gauss–Hermite quadrature; the default |
| <a href="#">mcaghermite</a> | mode-curvature adaptive Gauss–Hermite quadrature             |
| <a href="#">ghermite</a>    | nonadaptive Gauss–Hermite quadrature                         |

bootstrap, by, jackknife, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

startvalues(), noestimate, dnumerical, and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

### Model

`listwise` handles missing values through listwise deletion, which means that the entire observation is omitted from the estimation sample if any of the items are missing for that observation. By default, all nonmissing items in an observation are included in the likelihood calculation; only missing items are excluded.

### SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce\\_option](#).

### Reporting

`level(#)`; see [R] [estimation options](#).

`notable` suppresses the estimation table, either at estimation or upon replay.

`noheader` suppresses the output header, either at estimation or upon replay.

`display_options`: `nocl`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [estimation options](#).

### Integration

`intmethod(intmethod)` specifies the integration method to be used for computing the log likelihood. `mvaghermite` performs mean and variance adaptive Gauss–Hermite quadrature; `mcaghermite` performs mode and curvature adaptive Gauss–Hermite quadrature; and `ghermite` performs non-adaptive Gauss–Hermite quadrature.

The default integration method is `mvaghermite`.

`intpoints(#)` sets the number of integration points for quadrature. The default is `intpoints(7)`, which means that seven quadrature points are used to compute the log likelihood.

The more integration points, the more accurate the approximation to the log likelihood. However, computation time increases with the number of integration points.

### Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [maximize](#). Those that require special mention for `irt` are listed below.

`from()` accepts a properly labeled vector of initial values or a list of coefficient names with values. A list of values is not allowed.

The following options are available with `irt` but are not shown in the dialog box:

`startvalues()` specifies how starting values are to be computed. Starting values specified in `from()` override the computed starting values.

`startvalues(zero)` specifies that all starting values be set to 0. This option is typically useful only when specified with the `from()` option.

`startvalues(constantly)` builds on `startvalues(zero)` by fitting a constant-only model for each response to obtain estimates of intercept and cutpoint parameters.

`startvalues(fixedonly)` builds on `startvalues(constantly)` by fitting a full fixed-effects model for each response variable to obtain estimates of coefficients along with intercept and cutpoint parameters. You can also add suboption `iterate(#)` to limit the number of iterations `irt` allows for fitting the fixed-effects model.

`startvalues(ivloadings)` builds on `startvalues(fixedonly)` by using instrumental-variable methods with the generalized residuals from the fixed-effects models to compute starting values for latent-variable loadings. This is the default behavior.

`noestimate` specifies that the model is not to be fit. Instead, starting values are to be shown (as modified by the above options if modifications were made), and they are to be shown using the `coeflegend` style of output. An important use of this option is before you have modified starting values at all; you can type the following:

```
. irt ..., ... noestimate
. matrix b = e(b)
. ... (modify elements of b) ...
. irt ..., ... from(b)
```

`dnnumerical` specifies that during optimization, the gradient vector and Hessian matrix be computed using numerical techniques instead of analytical formulas. By default, `irt` uses analytical formulas for computing the gradient and Hessian for all integration methods.

`coeflegend`; see [R] [estimation options](#).

## Remarks and examples

[stata.com](http://www.stata.com)

The following discussion is about how to use `irt` to fit hybrid IRT models. In a hybrid model, one can fit different IRT models to subsets of items and perform a single calibration for the whole instrument. If you are new to the IRT features in Stata, we encourage you to read [IRT] `irt` first. If you are interested in the details of a specific IRT model, we refer you to the following.

### Binary response models

|                      |                                |
|----------------------|--------------------------------|
| <code>irt 1pl</code> | One-parameter logistic model   |
| <code>irt 2pl</code> | Two-parameter logistic model   |
| <code>irt 3pl</code> | Three-parameter logistic model |

### Categorical response models

|                      |                        |
|----------------------|------------------------|
| <code>irt grm</code> | Graded response model  |
| <code>irt nrm</code> | Nominal response model |
| <code>irt pcm</code> | Partial credit model   |
| <code>irt rsm</code> | Rating scale model     |

## ► Example 1: Combining an NRM and a PCM within a single instrument

In example 1 of [IRT] *irt nrm*, we applied a NRM to the physical science test data from de Ayala (2009). The last item is in fact an open-ended question scored on a scale of 1 to 4; thus, a PCM may be more appropriate for this item.

We fit an NRM to items q1–q3 and a PCM to item q4 as follows:

```
. use http://www.stata-press.com/data/r14/science
(Physical science data from de Ayala (2009))
. irt hybrid (nrm q1-q3) (pcm q4)
```

Fitting fixed-effects model:

```
Iteration 0: log likelihood = -9256.1514
Iteration 1: log likelihood = -9256.1514
```

Fitting full model:

```
Iteration 0: log likelihood = -9383.3438 (not concave)
Iteration 1: log likelihood = -9245.2232 (not concave)
Iteration 2: log likelihood = -9197.8737 (not concave)
Iteration 3: log likelihood = -9185.4678
Iteration 4: log likelihood = -9174.9401
Iteration 5: log likelihood = -9168.765
Iteration 6: log likelihood = -9168.2879
Iteration 7: log likelihood = -9168.2854
Iteration 8: log likelihood = -9168.2854
```

```
Hybrid IRT model          Number of obs      =      1,799
Log likelihood = -9168.2854
```

|            |         | Coef.     | Std. Err. | z     | P> z  | [95% Conf. Interval] |           |
|------------|---------|-----------|-----------|-------|-------|----------------------|-----------|
| <b>nrm</b> |         |           |           |       |       |                      |           |
| <b>q1</b>  |         |           |           |       |       |                      |           |
|            | Discrim |           |           |       |       |                      |           |
|            | 2 vs 1  | .4646847  | .1344317  | 3.46  | 0.001 | .2012034             | .728166   |
|            | 3 vs 1  | -.7170683 | .1835435  | -3.91 | 0.000 | -1.076807            | -.3573297 |
|            | 4 vs 1  | -.5973809 | .1601829  | -3.73 | 0.000 | -.9113337            | -.2834281 |
|            | Diff    |           |           |       |       |                      |           |
|            | 2 vs 1  | -.7692415 | .3102276  | -2.48 | 0.013 | -1.377276            | -.1612065 |
|            | 3 vs 1  | -1.137518 | .2353162  | -4.83 | 0.000 | -1.59873             | -.6763072 |
|            | 4 vs 1  | -.5488612 | .1512122  | -3.63 | 0.000 | -.8452317            | -.2524908 |
| <b>q2</b>  |         |           |           |       |       |                      |           |
|            | Discrim |           |           |       |       |                      |           |
|            | 2 vs 1  | -.1940619 | .2058633  | -0.94 | 0.346 | -.5975466            | .2094228  |
|            | 3 vs 1  | .555492   | .2001948  | 2.77  | 0.006 | .1631173             | .9478666  |
|            | 4 vs 1  | -.0926764 | .1957147  | -0.47 | 0.636 | -.4762703            | .2909174  |
|            | Diff    |           |           |       |       |                      |           |
|            | 2 vs 1  | 5.986222  | 6.587642  | 0.91  | 0.364 | -6.925319            | 18.89776  |
|            | 3 vs 1  | -3.52938  | 1.256797  | -2.81 | 0.005 | -5.992657            | -1.066102 |
|            | 4 vs 1  | 17.80327  | 37.97811  | 0.47  | 0.639 | -56.63246            | 92.239    |
| <b>q3</b>  |         |           |           |       |       |                      |           |
|            | Discrim |           |           |       |       |                      |           |
|            | 2 vs 1  | -.2375187 | .2286105  | -1.04 | 0.299 | -.6855871            | .2105497  |
|            | 3 vs 1  | .7013507  | .2196845  | 3.19  | 0.001 | .2707769             | 1.131924  |
|            | 4 vs 1  | 1.274853  | .2460029  | 5.18  | 0.000 | .7926961             | 1.75701   |
|            | Diff    |           |           |       |       |                      |           |
|            | 2 vs 1  | 1.128288  | 1.746526  | 0.65  | 0.518 | -2.29484             | 4.551415  |
|            | 3 vs 1  | -1.824877 | .4596289  | -3.97 | 0.000 | -2.725733            | -.9240207 |
|            | 4 vs 1  | -1.131442 | .1882568  | -6.01 | 0.000 | -1.500419            | -.7624657 |

| pcm    |                 |           |          |       |       |           |           |
|--------|-----------------|-----------|----------|-------|-------|-----------|-----------|
| q4     | Discrim<br>Diff | .3300807  | .0526185 | 6.27  | 0.000 | .2269504  | .4332111  |
| 2 vs 1 |                 | -2.056822 | .322035  | -6.39 | 0.000 | -2.687999 | -1.425645 |
| 3 vs 2 |                 | -.2976236 | .1923535 | -1.55 | 0.122 | -.6746295 | .0793823  |
| 4 vs 3 |                 | .8472732  | .2048051 | 4.14  | 0.000 | .4458626  | 1.248684  |

Note how the NRM and PCM are separated, so it is easy to tell which parameters correspond to which model. Because the PCM is nested in the NRM, we could perform a likelihood-ratio test to see whether our model is preferable to a pure NRM model; see [example 1](#) in [IRT] [irt](#) and [R] [lrtest](#) for more information.

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### ► Example 2: The 3PL model revisited

In [example 1](#) of [IRT] [irt 3pl](#), we used the mathematics and science data from [De Boeck and Wilson \(2004\)](#) to fit a 3PL model where the pseudoguessing parameter was constrained to be the same across items q1–q9. We mentioned that model identification problems can occur when one tries to estimate a separate guessing parameter for each item. In this example, we show how to deal with identification problems by constraining some pseudoguessing parameters to zero and fitting a 3PL model with separate guessing parameters to the remaining items.

We first fit a full 3PL model to items q1–q9, where each item has its own pseudoguessing parameter. Because the corresponding fixed-effects model is not identified, we limit the number of iterations `irt` spends fitting the fixed-effects model to 5.

```
. use http://www.stata-press.com/data/r14/masc1
(Data from De Boeck & Wilson (2004))
. irt 3pl q1-q9, seguessing startvalues(iterate(5))
Fitting fixed-effects model:
Iteration 0: log likelihood = -5322.8824
Iteration 1: log likelihood = -4291.3914 (not concave)
Iteration 2: log likelihood = -4270.0005 (not concave)
Iteration 3: log likelihood = -4269.7927 (not concave)
Iteration 4: log likelihood = -4269.7825 (not concave)
Iteration 5: log likelihood = -4269.7825 (not concave)
convergence not achieved
Fitting full model:
Iteration 0: log likelihood = -4227.4731 (not concave)
Iteration 1: log likelihood = -4188.8074 (not concave)
Iteration 2: log likelihood = -4134.829 (not concave)
Iteration 3: log likelihood = -4121.9664 (not concave)
Iteration 4: log likelihood = -4120.161 (not concave)
Iteration 5: log likelihood = -4119.33
Iteration 6: log likelihood = -4118.0626
Iteration 7: log likelihood = -4117.0488
Iteration 8: log likelihood = -4115.6541
Iteration 9: log likelihood = -4115.4168
Iteration 10: log likelihood = -4114.5522
Iteration 11: log likelihood = -4114.3738
Iteration 12: log likelihood = -4114.1039
Iteration 13: log likelihood = -4113.9668
Iteration 14: log likelihood = -4113.9036
Iteration 15: log likelihood = -4113.7972 (not concave)
Iteration 16: log likelihood = -4113.7712
Iteration 17: log likelihood = -4113.7505
```



We see that the pseudoguessing parameters for items q2, q3, q5, q8, and q9 are very close to zero. This suggests that we could fit a 2PL model to these five items and a full 3PL model with separate guessing parameters to the remaining four items.

```
. irt hybrid (2pl q2 q3 q5 q8 q9) (3pl q1 q4 q6 q7, sepg), startval(iter(5))
```

Fitting fixed-effects model:

```
Iteration 0: log likelihood = -4846.1954
Iteration 1: log likelihood = -4274.9988 (not concave)
Iteration 2: log likelihood = -4269.8038 (not concave)
Iteration 3: log likelihood = -4269.7889 (not concave)
Iteration 4: log likelihood = -4269.7825 (not concave)
Iteration 5: log likelihood = -4269.7825 (not concave)
convergence not achieved
```

Fitting full model:

```
Iteration 0: log likelihood = -4237.32 (not concave)
Iteration 1: log likelihood = -4156.6562
Iteration 2: log likelihood = -4122.4275
Iteration 3: log likelihood = -4115.0165
Iteration 4: log likelihood = -4113.7357
Iteration 5: log likelihood = -4113.7317
Iteration 6: log likelihood = -4113.7155 (not concave)
Iteration 7: log likelihood = -4113.7153
Iteration 8: log likelihood = -4113.7124
Iteration 9: log likelihood = -4113.7041
Iteration 10: log likelihood = -4113.6966
Iteration 11: log likelihood = -4113.6965
Iteration 12: log likelihood = -4113.6938
Iteration 13: log likelihood = -4113.694
Iteration 14: log likelihood = -4113.6933
Iteration 15: log likelihood = -4113.6933
```

```
Hybrid IRT model          Number of obs      =          800
Log likelihood = -4113.6933
```

|     |         | Coef.     | Std. Err. | z     | P> z  | [95% Conf. Interval] |           |
|-----|---------|-----------|-----------|-------|-------|----------------------|-----------|
| 2pl |         |           |           |       |       |                      |           |
| q2  | Discrim | .6595289  | .1141777  | 5.78  | 0.000 | .4357448             | .8833131  |
|     | Diff    | -.1491663 | .1200093  | -1.24 | 0.214 | -.3843802            | .0860477  |
| q3  | Discrim | 1.002143  | .1672015  | 5.99  | 0.000 | .6744346             | 1.329852  |
|     | Diff    | -1.611069 | .2184352  | -7.38 | 0.000 | -2.039194            | -1.182944 |
| q5  | Discrim | .8519284  | .1449869  | 5.88  | 0.000 | .5677594             | 1.136097  |
|     | Diff    | 1.65315   | .2444541  | 6.76  | 0.000 | 1.174029             | 2.132271  |
| q8  | Discrim | 1.477406  | .2581479  | 5.72  | 0.000 | .9714453             | 1.983366  |
|     | Diff    | -1.664009 | .1868519  | -8.91 | 0.000 | -2.030232            | -1.297786 |
| q9  | Discrim | .6233934  | .1200188  | 5.19  | 0.000 | .3881608             | .858626   |
|     | Diff    | -1.536899 | .286721   | -5.36 | 0.000 | -2.098862            | -.9749362 |



3pl

|    |         |           |          |       |       |           |          |
|----|---------|-----------|----------|-------|-------|-----------|----------|
| q1 | Discrim | 2.4122    | .868651  | 2.78  | 0.005 | .7096758  | 4.114725 |
|    | Diff    | -.0918963 | .2094073 | -0.44 | 0.661 | -.502327  | .3185344 |
|    | Guess   | .2054735  | .1066903 | 1.93  | 0.054 | -.0036357 | .4145828 |
| q4 | Discrim | 1.324646  | .5435556 | 2.44  | 0.015 | .2592968  | 2.389995 |
|    | Diff    | .8114595  | .2349801 | 3.45  | 0.001 | .350907   | 1.272012 |
|    | Guess   | .1921879  | .0961354 | 2.00  | 0.046 | .0037661  | .3806098 |
| q6 | Discrim | 2.160387  | .8880769 | 2.43  | 0.015 | .4197886  | 3.900986 |
|    | Diff    | .8876139  | .120097  | 7.39  | 0.000 | .652228   | 1.123    |
|    | Guess   | .1725445  | .0485985 | 3.55  | 0.000 | .0772932  | .2677957 |
| q7 | Discrim | .9441699  | 2.16823  | 0.44  | 0.663 | -3.305483 | 5.193823 |
|    | Diff    | 2.599752  | 1.781284 | 1.46  | 0.144 | -.8915002 | 6.091004 |
|    | Guess   | .1862199  | .210953  | 0.88  | 0.377 | -.2272403 | .5996801 |

Looking at the output, we see that the guessing parameters for q1, q4, q6, and q7 are similar. So we could further simplify this model by constraining the pseudoguessing parameter to be the same for the 3PL items. We could use a series of likelihood-ratio tests to choose among the competing models; see [example 1](#) in [\[IRT\] irt](#) and [\[R\] lrtest](#) for more information.

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## Stored results

irt stores the following in `e()`:

### Scalars

|                           |   |
|---------------------------|---|
| <code>e(N)</code>         | number of observations  |
| <code>e(k)</code>         | number of parameters  |
| <code>e(k_eq)</code>      | number of equations in <code>e(b)</code>                        |
| <code>e(k_dv)</code>      | number of dependent variables                                   |
| <code>e(k_rc)</code>      | number of covariances   |
| <code>e(k_rs)</code>      | number of variances   |
| <code>e(irt_k_eq)</code>  | number of IRT model groups                                      |
| <code>e(k_items#)</code>  | number of items in #th IRT model group                          |
| <code>e(sepguess#)</code> | 1 if #th IRT model contains a separate pseudoguessing parameter |
| <code>e(k_cat#)</code>    | number of categories for the #th item, ordinal                  |
| <code>e(k_out#)</code>    | number of outcomes for the #th item, nominal                    |
| <code>e(ll)</code>        | log likelihood  |
| <code>e(N_clust)</code>   | number of clusters  |
| <code>e(n_quad)</code>    | number of integration points                                    |
| <code>e(rank)</code>      | rank of <code>e(V)</code>                                       |
| <code>e(ic)</code>        | number of iterations  |
| <code>e(rc)</code>        | return code   |
| <code>e(converged)</code> | 1 if target model converged, 0 otherwise                        |

### Macros

|                         |   |
|-------------------------|---|
| <code>e(cmd)</code>     | <code>gsem</code>                         |
| <code>e(cmd2)</code>    | <code>irt</code>                          |
| <code>e(cmdline)</code> | command as typed                          |
| <code>e(model#)</code>  | name of IRT model for the #th model group |
| <code>e(items#)</code>  | names of items in #th IRT model group     |
| <code>e(n_cuts#)</code> | numlist of cuts in #th IRT model group    |

|                                   |  |
|-----------------------------------|--|
| <code>e(depvar)</code>            | names of all item variables  |
| <code>e(wtype)</code>             | weight type  |
| <code>e(wexp)</code>              | weight expression  |
| <code>e(title)</code>             | title in estimation output   |
| <code>e(clustvar)</code>          | name of cluster variable   |
| <code>e(family#)</code>           | family for the #th <i>item</i>   |
| <code>e(link#)</code>             | link for the #th <i>item</i>   |
| <code>e(intmethod)</code>         | integration method   |
| <code>e(vce)</code>               | <i>vcetype</i> specified in <code>vce()</code>                           |
| <code>e(vcetype)</code>           | title used to label Std. Err.  |
| <code>e(opt)</code>               | type of optimization   |
| <code>e(which)</code>             | max or min; whether optimizer is to perform maximization or minimization |
| <code>e(method)</code>            | estimation method: <code>ml</code>                                       |
| <code>e(ml_method)</code>         | type of <code>ml</code> method   |
| <code>e(user)</code>              | name of likelihood-evaluator program                                     |
| <code>e(technique)</code>         | maximization technique   |
| <code>e(datasignature)</code>     | the checksum   |
| <code>e(datasignaturevars)</code> | variables used in calculation of checksum                                |
| <code>e(properties)</code>        | <code>b V</code>   |
| <code>e(estat_cmd)</code>         | program used to implement <code>estat</code>                             |
| <code>e(predict)</code>           | program used to implement <code>predict</code>                           |
| <code>e(covariates)</code>        | list of covariates   |
| <code>e(footnote)</code>          | program used to implement the footnote display                           |

## Matrices

|                              |                                      |
|------------------------------|--------------------------------------|
| <code>e(_N)</code>           | sample size for each item            |
| <code>e(b)</code>            | parameter vector                     |
| <code>e(b_pclass)</code>     | parameter class                      |
| <code>e(cat#)</code>         | categories for the #th item, ordinal |
| <code>e(out#)</code>         | outcomes for the #th item, nominal   |
| <code>e(Cns)</code>          | constraints matrix                   |
| <code>e(ilog)</code>         | iteration log (up to 20 iterations)  |
| <code>e(gradient)</code>     | gradient vector                      |
| <code>e(V)</code>            | covariance matrix of the estimators  |
| <code>e(V_modelbased)</code> | model-based variance                 |

## Functions

|                        |                         |
|------------------------|-------------------------|
| <code>e(sample)</code> | marks estimation sample |
|------------------------|-------------------------|

## Methods and formulas

Let  $y_{ij}$  be the observed outcome for item  $i$  from person  $j$ . Define  $p_{ij} = \Pr(Y_{ij} = y_{ij} | \mathbf{B}_i, \theta_j)$ , where  $Y_{ij}$  represents the (yet to be observed) outcome,  $\mathbf{B}_i$  contains parameters for item  $i$ , and  $\theta_j$  is the ability (the latent trait) of person  $j$ . The functional form of  $p_{ij}$  and the parameters that go into  $\mathbf{B}_i$  depend on the choice of IRT model for item  $i$ .

Conditional on  $\theta_j$ , the item responses are assumed to be independent, so the conditional density for person  $j$  is given by

$$f(\mathbf{y}_j | \mathbf{B}, \theta_j) = \prod_{i=1}^I p_{ij}$$

where  $\mathbf{y}_j = (y_{1j}, \dots, y_{Ij})$ ,  $\mathbf{B} = (\mathbf{B}_1, \dots, \mathbf{B}_I)$ , and  $I$  is the number of items.

Missing items are skipped over in the above product by default. When the `listwise` option is specified, persons with any missing items are dropped from the estimation sample.

The likelihood for person  $j$  is computed by integrating out the latent variable from the joint density

$$L_j(\mathbf{B}) = \int_{-\infty}^{\infty} f(\mathbf{y}_j | \mathbf{B}, \theta_j) \phi(\theta_j) d\theta_j$$

where  $\phi(\cdot)$  is the density function for the standard normal distribution. The log likelihood for the estimation sample is simply the sum of the log likelihoods from the  $N$  persons in the estimation sample.

$$\log L(\mathbf{B}) = \sum_{j=1}^N \log L_j(\mathbf{B})$$

The integral in the formula for  $L_j(\mathbf{B})$  is generally not tractable, so we must use numerical methods.

## Gauss–Hermite quadrature

The integral of a function multiplied by the kernel of the standard normal distribution can be approximated using Gauss–Hermite quadrature (GHQ). For  $Q$ -point GHQ, let the abscissa and weight pairs be denoted by  $(x_q^*, w_q^*)$ ,  $q = 1, \dots, Q$ . The GHQ approximation is then

$$\int_{-\infty}^{\infty} f(x) \exp(-x^2) dx \approx \sum_{q=1}^Q w_q^* f(x_q^*)$$

Using the standard normal distribution yields the approximation

$$\int_{-\infty}^{\infty} f(x) \phi(x) dx \approx \sum_{q=1}^Q w_q f(x_q)$$

where  $x_q = \sqrt{2}x_q^*$  and  $w_q = w_q^*/\sqrt{\pi}$ . The GHQ approximation to the likelihood for person  $j$  is

$$L_j^{\text{GHQ}}(\mathbf{B}) = \sum_{q=1}^Q w_q f(\mathbf{y}_j | \mathbf{B}, x_q)$$

## Adaptive quadrature

This section sets the stage for mean–variance adaptive Gauss–Hermite quadrature (MVAGHQ) and mode–curvature adaptive Gauss–Hermite quadrature (MCAGHQ).

If we fix the item variables and the model parameters, we see that the posterior density for  $\theta_j$  is proportional to

$$\phi(\theta_j) f(\mathbf{y}_j | \mathbf{B}, \theta_j)$$

It is reasonable to assume that this posterior density can be approximated by a normal density with mean  $\mu_j$  and variance  $\tau_j$ . Instead of using the prior density of  $\theta_j$  as the weighting distribution in the integral, we can use our approximation for the posterior density,

$$L_j(\mathbf{B}) = \int_{-\infty}^{\infty} \frac{f(\mathbf{y}_j | \mathbf{B}, \theta_j) \phi(\theta_j)}{\phi(\theta_j, \mu_j, \tau_j)} \phi(\theta_j, \mu_j, \tau_j) d\theta_j$$

The likelihood is then approximated with

$$L_j^*(\mathbf{B}) = \sum_{q=1}^Q \omega_q f(\mathbf{y}_j | \mathbf{B}, \xi_q)$$

where  $\xi_q$  and the  $\omega_q$  are functions of  $x_q$  and  $w_q$  and the adaptive parameters  $\mu_j$  and  $\tau_j$ .

For MVAGHQ,  $\mu_j$  is the posterior mean, and  $\tau_j$  is the posterior variance of  $\theta_j$ . They are computed iteratively by updating the posterior moments by using the MVAGHQ approximation, starting with a zero mean and unit variance.

For MCAGHQ,  $\mu_j$  is the posterior mode for  $\theta_j$ , and  $\tau_j$  is the curvature at the mode. They are computed by optimizing the joint density with respect to  $\theta_j$ .

## References

- de Ayala, R. J. 2009. *The Theory and Practice of Item Response Theory*. New York: Guilford Press.
- De Boeck, P., and M. Wilson, ed. 2004. *Explanatory Item Response Models: A Generalized Linear and Nonlinear Approach*. New York: Springer.

## Also see

- [IRT] **irt hybrid postestimation** — Postestimation tools for irt hybrid
- [IRT] **irt** — Introduction to IRT models
- [IRT] **irt 1pl** — One-parameter logistic model
- [IRT] **irt 2pl** — Two-parameter logistic model
- [IRT] **irt 3pl** — Three-parameter logistic model
- [IRT] **irt grm** — Graded response model
- [IRT] **irt nrm** — Nominal response model
- [IRT] **irt pcm** — Partial credit model
- [IRT] **irt rsm** — Rating scale model
- [SEM] **gsem** — Generalized structural equation model estimation command
- [U] **20 Estimation and postestimation commands**